## 6689 <br> Edexcel GCE <br> Decision Mathematics D1 <br> (New Syllabus) <br> Advanced/Advanced Subsidiary <br> Monday 22 January 2001 - Afternoon <br> Time: 1 hour 30 minutes


#### Abstract

Materials required for examination Answer Book (AB16) Graph Paper (GP02) $\frac{\text { Items included with question papers }}{\text { Nil }}$

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such 48 G .


## Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexce), your centre number, candidate number, the unit title (Decision Mathematics D1), the paper reference (6689), your surname, other name and signature
When a calculator is used, the answer should be given to an appropriate degree of accuracy

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided
Full marks may be obtained for answers to ALL questions
This paper has 7 questions. Page 8 is blank

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answer without working may gain no credit.

1. This question should be answered on the sheet provided in the answer booklet.

A school wishes to link 6 computers. One is in the school office and one in each of rooms $A, B, C, D$ and $E$. Cables need to be laid to connect the computers. The school wishes to use a minimum total length of cable.

The table shows the shortest distances, in metres, between the various sites.

|  | Office | Room $A$ | Room $B$ | Room $C$ | Room $D$ | Room $E$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Office | - | 8 | 16 | 12 | 10 | 14 |
| Room $A$ | 8 | - | 14 | 13 | 11 | 9 |
| Room $B$ | 16 | 14 | - | 12 | 15 | 11 |
| Room $C$ | 12 | 13 | 12 | - | 11 | 8 |
| Room $D$ | 10 | 11 | 15 | 11 | - | 10 |
| Room $E$ | 14 | 9 | 11 | 8 | 10 | - |

(a) Starting at the school office, use Prim's algorithm to find a minimum spanning tree. Indicate the order in which you select the edges and draw you final tree.
(5 marks)
(b) Using your answer to part (a), calculate the minimum total length of cable required.
2. (a) Use the binary search algorithm to locate the name HUSSAIN in the following alphabetical list. Explain each step of the algorithm

1. ALLEN
2. BALL
. COOPER
3. EVANS
4. HUSSAIN
5. JONES
6. MICHAEL
7. PATEL
8. RICHARDS
9. TINDALL
10. $W U$
(b) State the maximum number of comparisons that need to be made to locate a name in an alphabetical list of 11 names.
11. 



Fig. 1
(a) Using an appropriate algorithm, obtain a suitable route starting and finishing at $A$.
(b) Calculate the total length of this route.
4. This question should be answered on the sheet provided in the answer booklet.

A manager has five workers, Mr. Ahmed, Miss Brown, Ms. Clough, Mr. Dingle and Mrs. Evans. To finish an urgent order he needs each of them to work overtime, one on each evening, in the next week. The workers are only available on the following evenings:

Mr. Ahmed (A) - Monday and Wednesday;
Miss Brown (B) - Monday, Wednesday and Friday; Ms. Clough (C) - Monday;
Mr. Dingle (D) - Tuesday, Wednesday and Thursday;
Mrs. Evans ( $E$ ) - Wednesday and Thursday.
The manager initially suggests that $A$ might work on Monday, $B$ on Wednesday and $D$ on Thursday.
(a) Using the nodes printed on the answer sheet, draw a bipartite graph to mode the availability of the five workers. Indicate, in a distinctive way, the manager's initial suggestion.
(b) Obtain an alternating path, starting at $C$, and use this to improve the initia matching.
(3 marks)
(c) Find another alternating path and hence obtain a complete matching.
5. This question should be answered on the sheet provided in the answer booklet.


Fig. 2

Figure 2 shows the activity network used to model a small building project. The activities are represented by the edges and the number in brackets on each edge represents the time, in hours, taken to complete that activity.
(a) Calculate the early time and the late time for each event. Write your answers in the boxes on the answer sheet.
(b) Hence determine the critical activities and the length of the critical path.

Each activity requires one worker. The project is to be completed in the minimum time.
(c) Schedule the activities for the minimum number of workers using the time line on the answer sheet. Ensure that you make clear the order in which each worker undertakes his activities.
6. This question should be answered on the sheet provided in the answer booklet.


Fig. 3
Figure 3 shows a capacitated, directed network. The number on each arc indicates the capacity of that arc.
(a) State the maximum flow along
(i) SAET, (ii) SBDT, (iii) SCFT.
(b) Show these maximum flows on Diagram 1 on the answer sheet.
(c) Taking your answer to part (b) as the initial flow pattern, use the labelling procedure to find a maximum flow from $S$ to $T$. Your working should be shown on Diagram 2. List each flow augmenting route you find, together with its flow.
(d) Indicate a maximum flow on Diagram 3.
(e) Prove that your flow is maximal.
7. A tailor makes two types of garment, $A$ and $B$. He has available $70 \mathrm{~m}^{2}$ of cotton fabric and $90 \mathrm{~m}^{2}$ of woollen fabric. Garment $A$ requires $1 \mathrm{~m}^{2}$ of cotton fabric and $3 \mathrm{~m}^{2}$ of woollen fabric. Garment $B$ requires $2 \mathrm{~m}^{2}$ of each fabric.

The tailor makes $x$ garments of type $A$ and $y$ garments of type $B$
(a) Explain why this can be modelled by the inequalities

$$
\begin{aligned}
& x+2 y \leq 70 \\
& 3 x+2 y \leq 90 \\
& x \geq 0, y \geq 0
\end{aligned}
$$

The tailor sells type $A$ for $£ 30$ and type $B$ for $£ 40$. All garments made are sold. The tailor wishes to maximise his total income.
(b) Set up an initial Simplex tableau for this problem.
(3 marks)
(c) Solve the problem using the Simplex algorithm.
(8 marks)
Figure 4 shows a graphical representation of the feasible region for this problem

(d) Obtain the coordinates of the points A, $C$ and $D$.
(4 marks)
(e) Relate each stage of the Simplex algorithm to the corresponding point in Fig. 4.
(3 marks)

## 6689 <br> Edexcel GCE <br> Decision Mathematics D1 <br> (New Syllabus) <br> Advanced/Advanced Subsidiary <br> Monday 25 June 2001 - Morning <br> Time: 1 hour 30 minutes

Materials required for examination
Answer Book (ABI2)
Graph Paper (ASG2)
Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculator such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

## nstructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Decision Mathematics D1), the paper reference (6689), your surname, other name and signature.

Information for Candidates
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled
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1. The precedence table for activities involved in a small project is shown below

| Activity | Preceding Activities |
| :---: | :---: |
| $A$ | - |
| $B$ | - |
| $C$ | - |
| $D$ | $B$ |
| $E$ | $A$ |
| $F$ | $A$ |
| $G$ | $B$ |
| $H$ | $C, D$ |
| $I$ | $E$ |
| $J$ | $E$ |
| $K$ | $F, G, I$ |
| $L$ | $H, J, K$ |

Draw an activity network, using activity on edge and without using dummies, to model this project.
2.


Figure 1 shows 7 locations $A, B, C, D, E, F$ and $G$ which are to be connected by pipelines. The arcs show the possible routes. The number on each arc gives the cost, in thousands of pounds, of laying that particular section.
(a) Use Kruskal's algorithm to obtain a minimum spanning tree for the network, giving the order in which you selected the arcs.
(b) Draw your minimum spanning tree and find the least cost of the pipelines.
3.

## Figure 2



Figure 2 shows a new small business park. The vertices $A, B, C, D, E, F$ and $G$ represent the various buildings and the arcs represent footpaths. The number on an arc gives the length, in metres, of the path. The management wishes to inspect each path to make sure it is fit for use.

Starting and finishing at $A$, solve the Route Inspection (Chinese Postman) problem for the network shown in Fig. 2 and hence determine the minimum distance thet needs to be walked in carrying out this inspection. Make your method and working clear and give a possible route of minimum length.
4. This question should be answered on the sheet provided in the answer booklet.


The weighted network shown in Fig. 3 models the area in which Bill lives. Each vertex represents a town. The edges represent the roads between the towns. The weights are the lengths, in km , of the roads.
(a) Use Dijkstra's algorithm to find the shortest route from Bill's home at $S$ to $T$. Complete all the boxes on the answer sheet and explain clearly how you determined the path of least weight from your labelling.
Bill decides that on the way to $T$ he must visit a shop in town $E$.
(b) Obtain his shortest route now, giving its length and explaining your method clearly.
(a) Use the bubble sort algorithm to sort the list of numbers above into descending order showing the rearranged order after each pass.
essica wants to record a number of television programmes onto video tapes Each tape is 2 hours long. The lengths, in minutes, of the programmes she wishes to record are

$$
55,45,20,30,30,40,20,90,25,50,35 \text { and } 35 .
$$

(b) Find the total length of programmes to be recorded and hence determine a lower bound for the number of tapes required.
(c) Use the first fit decreasing algorithm to fit the programmes onto her 2-hour tapes

Jessica's friend Amy says she can fit all the programmes onto 4 tapes
(d) Show how this is possible.
6. This question is to be answered on the sheet provided in the answer booklet.

Figure 4


Figure 4 shows a capacitated network. The numbers on each arc indicate the capacity of that arc in appropriate units.
(a) Explain why it is not possible to achieve a flow of 30 through the network from $S$ to $T$.
(b) State the maximum flow along
(i) SABT
(ii) $S C E T$
(c) Show these flows on Diagram 1 of the answer sheet.
(d) Taking your answer to part (c) as the initial flow pattern, use the labelling procedure to find the maximum flow from $S$ to $T$. Show your working on Diagram 2. List each flow-augmenting path you use together with its flow.
(e) Indicate a maximum flow on Diagram 3.
(f) Prove that your flow is maximal.
7. This question is to be answered on the sheet provided in the answer booklet.

A chemical company makes 3 products $X, Y$ and $Z$. It wishes to maximise its profit $£ P$. The manager considers the limitations on the raw materials available and models the situation with the following Linear Programming problem.

$$
\begin{array}{ll}
\text { Maximise } & P=3 x+6 y+4 z, \\
\text { subject to } & x \quad z \leq 4, \\
& x+4 y+2 z \leq 6, \\
& x+y+2 z \leq 12, \\
& x \geq 0, \quad y \geq 0, \quad z \geq 0,
\end{array}
$$

where $x, y$ and $z$ are the weights, in kg , of products $X, Y$ and $Z$ respectively.
A possible initial tableau is

| Basic variable | $x$ | $y$ | $z$ | $r$ | $s$ | $t$ | Value |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $r$ | 1 | 0 | 1 | 1 | 0 | 0 | 4 |
| $s$ | 1 | 4 | 2 | 0 | 1 | 0 | 6 |
| $t$ | 1 | 1 | 2 | 0 | 0 | 1 | 12 |
| $P$ | -3 | -6 | -4 | 0 | 0 | 0 | 0 |

(a) Explain
(i) the purpose of the variables $r, s$ and $t$,
(ii) the final row of the tableau.
(b) Solve this Linear Programming problem by using the Simplex alogorithm. Increase $y$ for your first iteration and than increase $x$ for your second iteration.
(c) Interpret your solution.

## 6689 <br> Edexcel GCE <br> Decision Mathematics D1 <br> (New Syllabus) <br> Advanced/Advanced Subsidiary <br> Friday 18 January 2002 - Afternoon <br> Time: 1 hour 30 minutes

Materials required for examination Graph Paper (ASG2)

## Items included with question paper

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48 G

## Instructions to Candidates

In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.

## nformation for Candidates

Full marks may be obtained for answers to ALL questions.
This paper has seven questions.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
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1. Ann, Bryn, Daljit, Gareth and Nickos have all joined a new committee. Each of them is to be allocated to one of five jobs $1,2,3,4$ or 5 . The table shows each member's preferences for the jobs.

| Ann | 1 or 2 |
| :---: | :---: |
| Bryn | 3 or 1 |
| Daljit | 2 or 4 |
| Gareth | 5 or 3 |
| Nickos | 1 or 2 |

Initially Ann, Bryn, Daljit and Gareth are allocated the first job in their lists shown in the table.
(a) Draw a bipartite graph to model the preferences shown in the table and indicate, in a distinctive way, the initial allocation of jobs.
(b) Use the matching improvement algorithm to find a complete matching, showing clearly your alternating path.
(c) Find a second alternating path from the initial allocation.
2. (i) Use the binary search algorithm to try to locate the name SABINE in the following alphabetical list. Explain each step of the algorithm.

$$
\begin{array}{ll}
\text { 1. } & \text { ABLE } \\
\text { 2. } & \text { BROWN } \\
\text { 3. } & \text { COOKE } \\
\text { 4. } & \text { DANIEL } \\
\text { 5. } & \text { DOUBLE } \\
\text { 6. } & \text { FEW } \\
\text { 7. } & \text { OSBORNE } \\
\text { 8. } & \text { PAUL } \\
\text { 9. } & \text { SWIFT } \\
\text { 10. } & \text { TURNER }
\end{array}
$$

(ii) Find the maximum number of iterations of the binary search algorithm needed to locate a name in a list of 1000 names
3. (i)

|  | $A$ | $B$ | $C$ | $D$ | $E$ | $F$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $A$ | - | 10 | 12 | 13 | 20 | 9 |
| $B$ | 10 | - | 7 | 15 | 11 | 7 |
| $C$ | 12 | 7 | - | 11 | 18 | 3 |
| $D$ | 13 | 15 | 11 | - | 27 | 8 |
| $E$ | 20 | 11 | 18 | 27 | - | 18 |
| $F$ | 9 | 7 | 3 | 8 | 18 | - |

The table shows the distances, in metres, between six nodes $A, B, C, D, E$, and $F$ of a network.
(a) Use Prim's algorithm, starting at $A$, to solve the minimum connector problem for this table of distances. Explain your method and indicate the order in which you selected the edges.
(b) Draw your minimum spanning tree and find its total length.
(c) State whether your minimum spanning tree is unique. Justify your answer.
(ii) A connected network $N$ has seven vertices.
(a) State the number of edges in a minimum spanning tree for $N$.

A minimum spanning tree for a connected network has $n$ edges.
(b) State the number of vertices in the network.
4.

## Figure 1



Figure 1 shows a network of roads. Erica wishes to travel from $A$ to $L$ as quickly as possible. The number on each edge gives the time, in minutes, to travel along that road.
(a) Use Dijkstra's algorithm to find a quickest route from $A$ to $L$. Complete all the boxes on the answer sheet and explain clearly how you determined the quickest route from your labelling.
(b) Show that there is another route which also takes the minimum time
5. Two fertilizers are available, a liquid $X$ and a powder $Y$. A bottle of $X$ contains 5 units of chemical $A, 2$ units of chemical $B$ and $\frac{1}{2}$ unit of chemical $C$. A packet of $Y$ contains 1 unit of $A, 2$ units of $B$ and 2 units of $C$. A professional gardener makes her own fertilizer. She requires at least 10 units of $A$, at least 12 units of $B$ and at least 6 units of $C$.

She buys $x$ bottles of $X$ and $y$ packets of $Y$.
(a) Write down the inequalities which model this situation.
(b) On the grid provided construct and label the feasible region.

A bottle of $X$ costs $£ 2$ and a packet of $Y$ costs $£ 3$.
(c) Write down an expression, in terms of $x$ and $y$, for the total cost $£ T$.
(d) Using your graph, obtain the values of $x$ and $y$ that give the minimum value of $T$. Make your method clear and calculate the minimum value of $T$.
(e) Suggest how the situation might be changed so that it could no longer be represented graphically


A company has 3 warehouses $W_{1}, W_{2}$, and $W_{3}$. It needs to transport the goods stored there to 2 retail outlets $R_{1}$ and $R_{2}$. The capacities of the possible routes, in van loads per day, are shown in Fig 2. Warehouses $W_{1}, W_{2}$ and $W_{3}$ have 14, 12 and 14 van loads respectively available per day and retail outlets $R_{1}$ and $R_{2}$ can accept 6 and 25 van loads respectively per day.
(a) On Diagram 1 on the answer sheet add a supersource $W$, a supersink $R$ and the appropriate directed arcs to obtain a single-source, single-sink capacitated network. State the minimum capacity of each arc you have added.
(b) State the maximum flow along

$$
\text { (i) } W W_{1} A R_{1} R, \quad \text { (ii) } W W_{3} C R_{2} R \text {. }
$$ (c) Taking your answers to part (b) as the initial flow pattern, use the labelling procedure to obtain a maximum flow through the network from $W$ to $R$. Show your working on Diagram 2. List each flow-augmenting route you use, together with its flow.

(d) From your final flow pattern, determine the number of van loads passing through $B$ each day.
The company has the opportunity to increase the number of vans loads from one of the warehouses $W_{1}, W_{2}, W_{3}$, to $A, B$ or $C$.
(e) Determine how the company should use this opportunity so that it achieves a maximal flow.
7.
Figure 3


A project is modelled by the activity network shown in Fig 3. The activities are represented by the edges. The number in brackets on each edge gives the time, in days, taken to complete the activity
(a) Calculate the early time and the late time for each event. Write these in the boxes on the answer sheet
(b) Hence determine the critical activities and the length of the critical path.
(c) Obtain the total float for each of the non-critical activities.
(d) On the first grid on the answer sheet, draw a cascade (Gantt) chart showing the information obtained in parts (b) and (c).

Each activity requires one worker. Only two workers are available.
(e) On the second grid on the answer sheet, draw up a schedule and find the minimum time in which the 2 workers can complete the project.

## 6689 <br> Edexcel GCE

Decision Mathematics D1
(New Syllabus)
Advanced/Advanced Subsidiary
Thursday 23 May 2002 - Afternoon
Time: 1 hour 30 minutes
$\frac{\text { Materials required for examination }}{\mathrm{Nil}}$
Items included with question papers Answer booklet

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates must NOT use
calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G

## Instructions to Candidates

In the boxes on the answer book, write your centre number, candidate number, your surname,
When a calculator is used, the answer should be given to an appropriate degree of accuracy
Witils and

## Information for Candidates

Full marks may be obtained for answers to ALL questions.
This paper has eight questions.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.
1.
.

| Ashford | 6 |
| :--- | :---: |
| Colnbrook | 1 |
| Datchet | 18 |
| Feltham | 12 |
| Halliford | 9 |
| Laleham | 0 |
| Poyle | 5 |
| Staines | 13 |
| Wraysbury | 14 |

The table above shows the points obtained by each of the teams in a football league after they had each played 6 games. The teams are listed in alphabetical order. Carry out a quick sort to produce a list of teams in descending order of points obtained.
(5)
2. While solving a maximizing linear programming problem, the following tableau was obtained.

| Basic <br> variable | $x$ | $y$ | $z$ | $r$ | $s$ | $t$ | Value |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $r$ | 0 | 0 | $1 \frac{2}{3}$ | 1 | 0 | $-\frac{1}{6}$ | $\frac{2}{3}$ |
| $y$ | 0 | 1 | $3 \frac{1}{3}$ | 0 | 1 | $-\frac{1}{3}$ | $\frac{1}{3}$ |
| $x$ | 1 | 0 | -3 | 0 | -1 | $\frac{1}{2}$ | 1 |
| $P$ | 0 | 0 | 1 | 0 | 1 | 1 | 11 |

(a) Explain why this is an optimal tableau.
(b) Write down the optimal solution of this problem, stating the value of every variable.
(c) Write down the profit equation from the tableau. Use it to explain why changing the value of any of the non-basic variables will decrease the value of $P$
3.


Five members of staff 1, 2, 3, 4 and 5 are to be matched to five jobs $A, B, C, D$ and $E$ A bipartite graph showing the possible matchings is given in Fig. 1 and an initial matching $M$ is given in Fig. 2

There are several distinct alternating paths that can be generated from $M$. Two such paths are

$$
\begin{aligned}
2-B & =4-E \\
\text { and } \quad 2-A & =3-D=5-E
\end{aligned}
$$

(a) Use each of these two alternating paths, in turn, to write down the complete matchings they generate.

Using the maximum matching algorithm and the initial matching $M$,
(b) find two further distinct alternating paths, making your reasoning clear.
4.


Figure 3 shows the network of paths in a country park. The number on each path gives its length in km . The vertices $A$ and $I$ represent the two gates in the park and the vertices $B, C, D, E, F, G$ and $H$ represent places of interest.
(a) Use Dijkstra's algorithm to find the shortest route from A to $I$. Show all necessary working in the boxes in the answer booklet and state your shortest route and its length.

The park warden wishes to check each of the paths to check for frost damage. She has to cycle along each path at least once, starting and finishing at $A$.
(b) (i) Use an appropriate algorithm to find which paths will be covered twice and state these paths.
(ii) Find a route of minimum length
(iii) Find the total length of this shortest route.
5. An algorithm is described by the flow chart below.

(a) Given that $a=645$ and $b=255$, complete the table in the answer booklet to show the results obtained at each step when the algorithm is applied.
(b) Explain how your solution to part (a) would be different if you had been given that $a=255$ and $b=645$.
(c) State what the algorithm achieves
6.

Figure 4


A building project is modelled by the activity network shown in Fig. 4. The activities are represented by the arcs. The number in brackets on each arc gives the time, in hours, taken to complete the activity. The left box entry at each vertex is the earliest event time and the right box entry is the latest event time.
(a) Determine the critical activities and state the length of the critical path.
(b) State the total float for each non-critical activity
(c) On the grid in the answer booklet, draw a cascade (Gantt) chart for the project
(4)

Given that each activity requires one worker,
(d) draw up a schedule to determine the minimum number of workers required to complete the project in the critical time. State the minimum number of workers.
7. A company wishes to transport its products from 3 factories $F_{1}, F_{2}$ and $F_{3}$ to a single retail outlet $R$. The capacities of the possible routes, in van loads per day, are shown in Fig. 5.

## Figure 5


(a) On Diagram 1 in the answer booklet add a supersource $S$ to obtain a capacitated network with a single source and a single sink. State the minimum capacity of each arc you have added.
(b) (i) State the maximum flow along $S F_{1} A B R$ and $S F_{3} C R$.
(ii) Show these maximum flows on Diagram 2 in the answer booklet, using numbers in circles.

Taking your answer to part (b)(ii) as the initial flow pattern,
(c) (i) use the labelling procedure to find a maximum flow from $S$ to $R$ Your working should be shown on Diagram 3. List each flow-augmenting route you find together with its flow.
(ii) Prove that your final flow is maximal.
8. A chemical company produces two products $X$ and $Y$. Based on potential demand, the total production each week must be at least 380 gallons. A major customer's weekly order for 125 gallons of $Y$ must be satisfied.

Product $X$ requires 2 hours of processing time for each gallon and product $Y$ requires 4 hours of processing time for each gallon. There are 1200 hours of processing time available each week. Let $x$ be the number of gallons of $X$ produced and $y$ be the number of gallons of $Y$ produced each week.
(a) Write down the inequalities that $x$ and $y$ must satisfy.

It costs $£ 3$ to produce 1 gallon of $X$ and $£ 2$ to produce 1 gallon of $Y$. Given that the total cost of production is $£ C$,
(b) express $C$ in terms of $x$ and $y$.
(1)

The company wishes to minimise the total cost.
(c) Using the graphical method, solve the resulting Linear Programming problem. Find the optimal values of $x$ and $y$ and the resulting total cost.
(d) Find the maximum cost of production for all possible choices of $x$ and $y$ which satisfy the inequalities you wrote down in part (a).

## 6689

## Edexcel GCE

Decision Mathematics D1
(New Syllabus)
Advanced/Advanced Subsidiary
Tuesday 5 November 2002 - Morning
Time: 1 hour 30 minutes
$\frac{\text { Materials required for examination }}{\text { Graph Paper (ASG2) } \quad \frac{\text { Items included with question papers }}{\text { Answer booklet }}}$

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algebra, differentiation and/or integration. Thus candidates must NOT use
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## Information for Candidates

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Advice to Candidates
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1.

Figure 1


A Hamilton cycle for the graph in Fig. 1 begins $A, X, D, V, \ldots$
(a) Complete this Hamiltonian cycle.
(b) Hence use the planarity algorithm to determine if the graph is planar.
2. The precedence table for activities involved in manufacturing a toy is shown below.

| Activity | Preceding activity |
| :---: | :---: |
| $A$ | - |
| $B$ | - |
| $C$ | - |
| $D$ | $A$ |
| $E$ | $A$ |
| $F$ | $B$ |
| $G$ | $B$ |
| $H$ | $C, D, E, F$ |
| $I$ | $E$ |
| $J$ | $E$ |
| $K$ | $I$ |
| $L$ | $I$ |
| $M$ | $G, H, K$ |

(a) Draw an activity network, using activity on arc, and exactly one dummy, to model the manufacturing process.
(b) Explain briefly why it is necessary to use a dummy in this case.
3. At a water sports centre there are five new instructors. Ali $(A)$, George $(G)$, Jo ( $J$ ), Lydia $(L)$ and Nadia ( $N$ ). They are to be matched to five sports, canoeing ( $C$ ), scuba diving $(D)$, surfing $(F)$, sailing $(S)$ and water skiing $(W)$

The table indicates the sports each new instructor is qualified to teach.

| Instructor | Sport |
| :---: | :---: |
| $A$ | $C, F, W$ |
| $G$ | $F$ |
| $J$ | $D, C, S$ |
| $L$ | $S, W$ |
| $N$ | $D, F$ |

Initially, $A, G, J$ and $L$ are each matched to the first sport in their individual list.
(a) Draw a bipartite graph to model this situation and indicate the initial matching in a distinctive way.
(b) Starting from this initial matching, use the maximum matching algorithm to find a complete matching. You must clearly list any alternating paths used.

Given that on a particular day $J$ must be matched to $D$
(c) explain why it is no longer possible to find a complete matching.
4.


Figure 2 models an underground network of pipes that must be inspected for leaks. The nodes $A$, $B, C, D, E, F, G$ and $H$ represent entry points to the network. The number on each arc gives the length, in metres, of the corresponding pipe.

Each pipe must be traversed at least once and the length of the inspection route must be minimised.
(a) Use the Route Inspection algorithm to find which paths, if any, need to be traversed twice

It is decided to start the inspection at node $C$. The inspection must still traverse each pipe at least once but may finish at any node.
(b) Explaining your reasoning briefly, determine the node at which the inspection should finish if the route is to be minimised. State the length of your route.
5.

(a) Use Dijkstra's algorithm to find the shortest route from $S$ to $T$ in Fig. 3. Show all necessary working in the boxes in the answer booklet. State your shortest route and its length.
(b) Explain how you determined the shortest route from your labelling.
(c) It is now necessary to go from $S$ to $T$ via $H$. Obtain the shortest route and its length.
6.
(a) The list of numbers above is to be sorted into descending order. Perform a bubble sort to obtain the sorted list, giving the state of the list after each complete pass.

The numbers in the list represent weights, in grams, of objects which are to be packed into bins that hold up to 100 g .
(b) Determine the least number of bins needed.
(c) Use the first-fit decreasing algorithm to fit the objects into bins which hold up to 100 g .
7.

Figure 4


The network in Fig. 4 models a drainage system. The number on each arc indicates the capacity of that arc, in litres per second.
(a) Write down the source vertices
(2)


Figure 5 shows a feasible flow through the same network
(b) State the value of the feasible flow shown in Fig. 5.

Taking the flow in Fig. 5 as your initial flow pattern,
(c) use the labelling procedure on Diagram 1 to find a maximum flow through this network. You should list each flow-augmenting route you use, together with its flow.
d) Show the maximal flow on Diagram 2 and state its value.
(e) Prove that your flow is maximal.
8. T42 Co. Ltd produces three different blends of tea, Morning, Afternoon and Evening. The teas must be processed, blended and then packed for distribution. The table below shows the time taken, in hours, for each stage of the production of a tonne of tea. It also shows the profit, in hundreds of pounds, on each tonne.

|  | Processing | Blending | Packing | Profit (£100) |
| :--- | :---: | :---: | :---: | :---: |
| Morning blend | 3 | 1 | 2 | 4 |
| Afternoon blend | 2 | 3 | 4 | 5 |
| Evening blend | 4 | 2 | 3 | 3 |

The total times available each week for processing, blending and packing are 35, 20 and 24 hours respectively. T42 Co. Ltd wishes to maximise the weekly profit.

Let $x, y$ and $z$ be the number of tonnes of Morning, Afternoon and Evening blend produced each week.
(a) Formulate the above situation as a linear programming problem, listing clearly the objective function, and the constraints as inequalities.

An initial Simplex tableau for the above situation is

| Basic <br> variable | $x$ | $y$ | $z$ | $r$ | $s$ | $t$ | Value |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $r$ | 3 | 2 | 4 | 1 | 0 | 0 | 35 |
| $s$ | 1 | 3 | 2 | 0 | 1 | 0 | 20 |
| $t$ | 2 | 4 | 3 | 0 | 0 | 1 | 24 |
| $P$ | -4 | -5 | -3 | 0 | 0 | 0 | 0 |

(b) Solve this linear programming problem using the Simplex algorithm. Take the most negative number in the profit row to indicate the pivot column at each stage.

T42 Co. Ltd wishes to increase its profit further and is prepared to increase the time available for processing or blending or packing or any two of these three.
(c) Use your answer to part (b) to advise the company as to which stage(s) it should increase the time available.

6689
Edexcel GCE
Decision Mathematics D1 Advanced/Advanced Subsidiary
Friday 17 January 2003 - Afternoon
Time: 1 hour $\mathbf{3 0}$ minutes

## Materials required for examination $\frac{\text { papers }}{\text { Nil }}$

Items included with question Answer booklet
andidates may use any calculator EXCEPT those with the facility for symbolic Igebra, differentiation and/or integration. Thus candidates must NOT us calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G Hewlett Packard HP 48G.

## Instructions to Candidates

In the boxes on the answer book, write your centre number, candidate number, your urname, initials and signature
When a calculator is used, the answer should be given to an appropriate degree of accuracy.
Information for Candidates
Full marks may be obtained for answers to ALL questions.
This paper has eight questions.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.
1.

Figure 1


Use the planarity algorithm to show that the graph in Fig. 1 is planar.
2. At Tesafe supermarket there are 5 trainee staff, Homan (H), Jenna ( $J$ ), Mary (M), Tim (T) and Yoshie ( $Y$ ). They each must spend one week in each of 5 departments, Delicatessen ( $D$ ) Frozen foods $(F)$, Groceries $(G)$, Pet foods $(P)$, Soft drinks $(S)$. Next week every department requires exactly one trainee. The table below shows the departments in which the trainees have yet to spend time.

| Trainee | Departments |
| :---: | :---: |
| $H$ | $D, F, P$ |
| $J$ | $G, D, F$ |
| $M$ | $S, P, G$ |
| $T$ | $F, S, G$ |
| $Y$ | $D$ |

Initially $H, J, M$ and $T$ are allocated to the first department in their list.
(a) Draw a bipartite graph to model this situation and indicate the initial matching in a distinctive way.

## Starting from this matching,

(b) use the maximum matching algorithm to find a complete matching. You must make clear your alternating path and your complete matching.
3. A manager wishes to purchase seats for a new cinema. He wishes to buy three types of seat; standard, deluxe and majestic. Let the number of standard, deluxe and majestic seats to be bought be $x, y$ and $z$ respectively.
He decides that the total number of deluxe and majestic seats should be at most half of the number of standard seats.
The number of deluxe seats should be at least $10 \%$ and at most $20 \%$ of the total number of The n
seats
The number of majestic seats should be at least half of the number of deluxe seats.
The total number of seats should be at least 250 .
Standard, deluxe and majestic seats each cost $£ 20, £ 26$ and $£ 36$, respectively The manager wishes to minimize the total cost, $£ C$, of the seats.

Formulate this situation as a linear programming problem, simplifying your inequalities so that all the coefficients are integers.

Figure 2


The arcs in Fig. 2 represent roads in a town. The weight on each arc gives the time, in minutes, taken to drive along that road. The times taken to drive along $A B$ and $D E$ vary depending upon the time of day.

A police officer wishes to drive along each road at least once, starting and finishing at $A$. The journey is to be completed in the least time.
(a) Briefly explain how you know that a route between $B$ and $E$ will have to be repeated.
(b) List the possible routes between $B$ and $E$. State how long each would take, in terms of $x$ where appropriate.
(c) Find the range of values that $x$ must satisfy so that $D E$ would be one of the repeated arcs.

Given that $x=7$,
(d) find the total time needed for the police officer to carry out this journey
5. Figure 3


| Key |  |
| :--- | :--- |
| Earliest <br> event time | Latest <br> event time |

A project is modelled by the activity network in Fig. 3. The activities are represented by the arcs. One worker is required for each activity. The number in brackets on each arc gives the time, in hours, to complete the activity. The earliest event time and the latest event time are given by the numbers in the left box and right box respectively.
(a) State the value of $x$ and the value of $y$.
(b) List the critical activities
(c) Explain why at least 3 workers will be needed to complete this project in 38 hours.
(d) Schedule the activities so that the project is completed in 38 hours using just 3 workers. You must make clear the start time and finish time of each activity.

The list of numbers above is to be sorted into descending order.
(a) (i) Perform the first pass of a bubble sort, giving the state of the list after each exchange.
(ii) Perform further passes, giving the state of the list after each pass, until the algorithm terminates.

The numbers represent the lengths, in cm , of pieces to be cut from rods of length 50 cm .
(b) (i) Show the result of applying the first fit decreasing bin packing algorithm to this situation.
(ii) Determine whether your solution to (b) (i) has used the minimum number of 50 cm rods.
7.

Figure 4


Figure 4 shows a capacitated directed network. The number on each arc is its capacity. The numbers in circles show a feasible flow from sources $A$ and $B$ to sinks $I, J$ and $K$.
Take this as the initial flow pattern.
(a) On Diagram 1 in the answer booklet, add a supersource $S$ and a supersink $W$ to obtain a capacitated network with a single source and single sink. State the minimum capacities of the arcs you have added.
(b) (i) Use the given initial flow and the labelling procedure on Diagram 2 to find the maximum flow through the network. You must list each flow-augmenting route you use together with its flow.
(ii) Verify that your flow is maximal.
(c) Show your maximum flow pattern on Diagram 3.
8. The tableau below is the initial tableau for a maximising linear programming problem.

| Basic Variable | $x$ | $y$ | $z$ | $r$ | $s$ | Value |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $r$ | 2 | 3 | 4 | 1 | 0 | 8 |
| $s$ | 3 | 3 | 1 | 0 | 1 | 10 |
| $P$ | -8 | -9 | -5 | 0 | 0 | 0 |

(a) For this problem $x \geq 0, y \geq 0, z \geq 0$. Write down the other two inequalities and the objective function.
(b) Solve this linear programming problem
(c) State the final value of $P$, the objective function, and of each of the variables.

## Edexcel GCE

## Decision Mathematics D1

Advanced/Advanced Subsidiary
Tuesday 10 June 2003 - Afternoon
Time: $\mathbf{1}$ hour 30 minutes

Materials required for examination Nil D1 Answer booklet

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates must NOT use calculator such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates
In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.

Information for Candidates
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.
Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

## Write your answers in the Dl answer booklet for this paper

1. Six workers $A, B, C, D, E$ and $F$ are to be matched to six tasks $1,2,3,4,5$ and 6 .

The table below shows the tasks that each worker is able to do.

| Worker | Tasks |
| :---: | :---: |
| $A$ | $2,3,5$ |
| $B$ | $1,3,4,5$ |
| $C$ | 2 |
| $D$ | 3,6 |
| $E$ | $2,4,5$ |
| $F$ | 1 |

A bipartite graph showing this information is drawn in the answer booklet.
Initially, $A, B, D$ and $E$ are allocated to tasks $2,1,3$ and 5 respectively.
Starting from the given initial matching, use the matching improvement algorithm to find a complete matching, showing your alternating paths clearly.
2. (a) Explain why it is impossible to draw a network with exactly three odd vertices.

## Figure 1



The Route Inspection problem is solved for the network in Fig. 1 and the length of the route is found to be 100 .
(b) Determine the value of $x$, showing your working clearly
3. (a) Describe the differences between Prim's algorithm and Kruskal's algorithm for finding a minimum connector of a network.

Figure 2

(b) Listing the arcs in the order that you select them, find a minimum connector for the network in Fig. 2, using
(i) Prim's algorithm,
(ii) Kruskal's algorithm.
4. The following list gives the names of some students who have represented Britain in the International Mathematics Olympiad

Roper (R), Palmer (P), Boase (B), Young (Y), Thomas (T), Kenney (K), Morris (M), Halliwell (H), Wicker ( $W$ ), Garesalingam ( $G$ ).
(a) Use the quick sort algorithm to sort the names above into alphabetical order
(b) Use the binary search algorithm to locate the name Kenney.
5.

Figure 3


The network in Fig. 3 shows the activities involved in the process of producing a perfume. The activities are represented by the arcs. The number in brackets on each arc gives the time, in hours, taken to complete the activity.
(a) Calculate the early time and the late time for each event, showing them on Diagram 1 in the answer booklet.
(b) Hence determine the critical activities.
(c) Calculate the total float time for $D$.

Each activity requires only one person.
(d) Find a lower bound for the number of workers needed to complete the process in the minimum time.

Given that there are only three workers available, and that workers may not share an activity,
(e) schedule the activities so that the process is completed in the shortest time. Use the time line in the answer booklet. State the new shortest time.
6. A company produces two types of self-assembly wooden bedroom suites, the 'Oxford' and the 'York'. After the pieces of wood have been cut and finished, all the materials have to be packaged. The table below shows the time, in hours, needed to complete each stage of the process and the profit made, in pounds, on each type of suite.

|  | Oxford | York |
| :--- | :---: | :---: |
| Cutting | 4 | 6 |
| Finishing | 3.5 | 4 |
| Packaging | 2 | 4 |
| Profit $(\mathfrak{£})$ | 300 | 500 |

The times available each week for cutting, finishing and packaging are 66,56 and 40 hours respectively.

The company wishes to maximise its profit.
Let $x$ be the number of Oxford, and $y$ be the number of York suites made each week.
(a) Write down the objective function.
(b) In addition to

$$
\begin{aligned}
2 x+3 y & \leq 33, \\
x & \geq 0, \\
y & \geq 0,
\end{aligned}
$$

find two further inequalities to model the company's situation.
(c) On the grid in the answer booklet, illustrate all the inequalities, indicating clearly the feasible region.
(d) Explain how you would locate the optimal point
(e) Determine the number of Oxford and York suites that should be made each week and the maximum profit gained.

It is noticed that when the optimal solution is adopted, the time needed for one of the three stages of the process is less than that available.
(f) Identify this stage and state by how many hours the time may be reduced
7.


Figure 4 shows a capacitated, directed network. The unbracketed number on each arc indicates the capacity of that arc, and the numbers in brackets show a feasible flow of value 68 through the network.
(a) Add a supersource and a supersink, and arcs of appropriate capacity, to Diagram 2 in the answer booklet
(b) Find the values of $x$ and $y$, explaining your method briefly
(c) Find the value of cuts $C_{1}$ and $C_{2}$.

Starting with the given feasible flow of 68 ,
(d) use the labelling procedure on Diagram 3 to find a maximal flow through this network. List each flow-augmenting route you use, together with its flow.
(e) Show your maximal flow on Diagram 4 and state its value.
(f) Prove that your flow is maximal.

## Edexcel GCE

## Decision Mathematics D1

Advanced/Advanced Subsidiary
Tuesday 4 November 2003 - Morning
Time: 1 hour 30 minutes
$\frac{\text { Materials required for examination }}{\text { Nil }}$ Nil

Items included with question papers D1 Answer booklet

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates must NOT use calculator such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

## Instructions to Candidates

In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature

## Information for Candidates

Full marks may be obtained for answers to ALL questions
This paper has eight questions

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit

## Write your answers in the Dl answer booklet for this paper

1. 



A local council is responsible for maintaining pavements in a district. The roads for which it is responsible are represented by arcs in Fig. 1.The junctions are labelled $A, B, C, \ldots, G$. The number on each arc represents the length of that road in km.

The council has received a number of complaints about the condition of the pavements. In order to inspect the pavements, a council employee needs to walk along each road twice (once on each side of the road) starting and ending at the council offices at $C$. The length of the route is to be minimal. Ignore the widths of the roads.
(a) Explain how this situation differs from the standard Route Inspection problem.
(b) Find a route of minimum length and state its length.
2. An electronics company makes a product that consists of components $A, B, C, D, E$ and $F$ The table shows which components must be connected together to make the product work The components are all placed on a circuit board and connected by wires, which are not allowed to cross.

| Component | Must be connected to |
| :---: | :---: |
| $A$ | $B, D, E, F$ |
| $B$ | $C, D, E$ |
| $C$ | $D, E$ |
| $D$ | $E$ |
| $E$ | $F$ |
| $F$ | $B$ |

(a) On the diagram in the answer book draw straight lines to show which components need to be connected.
(b) Starting with the Hamiltonian cycle $A B C D E F A$, use the planarity algorithm to determine whether it is possible to build this product on a circuit board.
3.

Figure 2


The bipartite graph in Fig. 2 shows the possible allocations of people $A, B, C, D, E$ and $F$ to tasks $1,2,3,4,5$ and 6.

An initial matching is obtained by matching the following pairs
A to 3,
$B$ to 4,
$C$ to 1,
$F$ to 5 .
(a) Show this matching in a distinctive way on the diagram in the answer book.
(b) Use an appropriate algorithm to find a maximal matching. You should state any alternating paths you have used.
4. (a) Draw an activity network described in this precedence table, using as few dummies as possible.

| Activity | Must be preceded by: |
| :---: | :---: |
| $A$ | - |
| $B$ | $A$ |
| $C$ | $A$ |
| $D$ | $A$ |
| $E$ | $C$ |
| $F$ | $C$ |
| $G$ | $B, D, E, F$ |
| $H$ | $B, D, E, F$ |
| $I$ | $F, D$ |
| $J$ | $F, D$ |
| $K$ | $K$ |
| $L$ |  |

(a) A different project is represented by the activity network shown in Fig. 3. The duration of each activity is shown in brackets.

Figure 3


Find the range of values of $x$ that will make $D$ a critical activity.
5. Nine pieces of wood are required to build a small cabinet. The lengths, in cm , of the pieces of wood are listed below

$$
20, \quad 20, \quad 20, \quad 35, \quad 40, \quad 50, \quad 60, \quad 70, \quad 75
$$

Planks, one metre in length, can be purchased at a cost of $£ 3$ each.
(a) The first fit decreasing algorithm is used to determine how many of these planks are to be purchased to make this cabinet. Find the total cost and the amount of wood wasted.

Planks of wood can also be bought in 1.5 m lengths, at a cost of $£ 4$ each. The cabinet can be built using a mixture of 1 m and 1.5 m planks.
(b) Find the minimum cost of making this cabinet. Justify your answer.
6. (a) Define the terms
(i) tree,
(ii) spanning tree,
(iii) minimum spanning tree
(b) State one difference between Kruskal's algorithm and Prim's algorithm, to find a minimum spanning tree.

(c) Use Kruskal's algorithm to find the minimum spanning tree for the network shown in Fig. 4. State the order in which you included the arcs. Draw the minimum spanning tree in Diagram 1 in the answer book and state its length.

## Figure 5



Figure 5 models a car park. Currently there are two pay-stations, one at $E$ and one at $N$. These two are linked by a cable as shown. New pay-stations are to be installed at $B, H, A, F$ and $C$ The number on each arc represents the distance between the pay-stations in metres. All of the pay-stations need to be connected by cables, either directly or indirectly. The current cable between $E$ and $N$ must be included in the final network. The minimum amount of new cable is to be used.
(d) Using your answer to part (c), or otherwise, determine the minimum amount of new cable needed. Use Diagram 2 to show where these cables should be installed. State the minimum amount of new cable needed
7.

## Figure 6



Figure 6 shows a capacitated, directed network of pipes flowing from two oil fields $F_{1}$ and $F_{2}$ to three refineries $R_{1}, R_{2}$ and $R_{3}$. The number on each arc represents the capacity of the pipe and the numbers in the circles represent a possible flow of 65 .
(a) Find the value of $x$ and the value of $y$
(b) On Diagram 1 in the answer book, add a supersource and a supersink, and arcs showing their minimum capacities.
(c) Taking the given flow of 65 as the initial flow pattern, use the labelling procedure on Diagram 2 to find the maximum flow. State clearly your flow augmenting routes.
(d) Show the maximum flow on Diagram 3 and write down its value.
(e) Verify that this is the maximum flow by finding a cut equal to the flow.
8. A company makes three sizes of lamps, small, medium and large. The company is trying to determine how many of each size to make in a day, in order to maximise its profit. As part of the process the lamps need to be sanded, painted, dried and polished. A single machine carries out these tasks and is available 24 hours per day. A small lamp requires one hour on this machine, a medium lamp 2 hours and a large lamp 4 hours.

Let $x=$ number of small lamps made per day,

$$
y=\text { number of medium lamps made per day, }
$$

$z=$ number of large lamps made per day,
where $x \geq 0, y \geq 0$ and $z \geq 0$.
(a) Write the information about this machine as a constraint.
(b) (i) Re-write your constraint from part (a) using a slack variable $s$.
(ii) Explain what $s$ means in practical terms.

Another constraint and the objective function give the following Simplex tableau. The profit $P$ is stated in euros.

| Basic variable | $x$ | $y$ | $z$ | $r$ | $s$ | Value |
| :---: | ---: | ---: | ---: | ---: | ---: | :---: |
| $r$ | 3 | 5 | 6 | 1 | 0 | 50 |
| $s$ | 1 | 2 | 4 | 0 | 1 | 24 |
| $P$ | -1 | -3 | -4 | 0 | 0 | 0 |

(c) Write down the profit on each small lamp.
(d) Use the Simplex algorithm to solve this linear programming problem.
(e) Explain why the solution to part (d) is not practical.
(f) Find a practical solution which gives a profit of 30 euros. Verify that it is feasible.

## 6689 <br> Edexcel GCE <br> Decision Mathematics D1 <br> Advanced/Advanced Subsidiary <br> Friday 16 January 2004 - Afternoon <br> Time: 1 hour 30 minutes <br> Materials required for examination

Candidates may use any calculator EXCEPT those with the faciity for symbolic algebra, differentiation and/or integration. Thus candidates must NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

## Instructions to Candidates

In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidates

Full marks may be obtained for answers to ALL questions.
This paper has eight questions

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit

1. Define the terms
(a) bipartite graph,
(b) alternating path,
(c) matching,
(d) complete matching.
2. A three-variable linear programming problem in $x, y$ and $z$ is to be solved. The objective is to maximise the profit P . The following tableau was obtained.

| Basic variable | $x$ | $y$ | $z$ | $r$ | $s$ | $t$ | Value |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $s$ | 3 | 0 | 2 | 0 | 1 | $-\frac{2}{3}$ | $\frac{2}{3}$ |
| $r$ | 4 | 0 | $\frac{7}{2}$ | 1 | 0 | 8 | $\frac{9}{2}$ |
| $y$ | 5 | 1 | 7 | 0 | 0 | 3 | 7 |
| $P$ | 3 | 0 | 2 | 0 | 0 | 8 | 63 |

(a) State, giving your reason, whether this tableau represents the optimal solution.
(b) State the values of every variable.
(c) Calculate the profit made on each unit of $y$.
3.

Figure 1
4.

Figure 2


An engineer needs to check the state of a number of roads to see whether they need esurfacing. The roads that need to be checked are represented by the arcs in Fig 2. Th number on each arc represents the length of that road in km . To check all the roads, he need to travel along each road at least once. He wishes to minimise the total distance travelled.

The engineer's office is at $G$, so he starts and ends his journey at $G$
a) Use an appropriate algorithm to find a route for the engineer to follow. State your rout and its length.

The engineer lives at $D$. He believes he can reduce the distance travelled by starting from home and inspecting all the roads on the way to his office at $G$.
b) State whether the engineer is correct in his belief. If so, calculate how much shorter his new route is. If not, explain why not
(a) Find the capacity of each of the three cuts.
(b) Verify that the flow of 26 is maximal

The government aims to maximise the possible flow from $B$ to $L$ by using one of two options.
Option 1: Build a new road from $E$ to $J$ with capacity 5.
or Option 2: Build a new road from $F$ to $H$ with capacity 3
(c) By considering both options, explain which one meets the government's aim
5.

## Figure 3



Figure 3 describes an algorithm in the form of a flow chart, where $a$ is a positive integer.
List $P$, which is referred to in the flow chart, comprises the prime numbers $2,3,5,7,11,13$, $17, \ldots$
(a) Starting with $a=90$, implement this algorithm. Show your working in the table in the answer book.
(b) Explain the significance of the output list.
(c) Write down the final value of $c$ for any initial value of $a$.
6.

|  | $A$ | $B$ | $C$ | $D$ | $E$ | $F$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $A$ | - | 7 | 3 | - | 8 | 11 |
| $B$ | 7 | - | 4 | 2 | - | 7 |
| $C$ | 3 | 4 | - | 5 | 9 | - |
| $D$ | - | 2 | 5 | - | 6 | 3 |
| $E$ | 8 | - | 9 | 6 | - | - |
| $F$ | 11 | 7 | - | 3 | - | - |

The matrix represents a network of roads between six villages $A, B, C, D, E$ and $F$. The value in each cell represents the distance, in km, along these roads.
(a) Show this information on the diagram in the answer book
(b) Use Kruskal's algorithm to determine the minimum spanning tree. State the order in which you include the arcs and the length of the minimum spanning tree. Draw the minimum spanning tree.
(c) Starting at $D$, use Prim's algorithm on the matrix given in the answer book to find the minimum spanning tree. State the order in which you include the arcs.
7. Becky's bird food company makes two types of bird food. One type is for bird feeders and the other for bird tables. Let $x$ represent the quantity of food made for bird feeders and $y$ represent the quantity of food made for bird tables. Due to restrictions in the production process, and known demand, the following constraints apply.

$$
\begin{gathered}
x+y \leq 12, \\
y<2 x, \\
2 y \geq 7, \\
y+3 x \geq 15 .
\end{gathered}
$$

(a) On the axes provided, show these constraints and label the feasible region $R$.

The objective is to minimise $C=2 x+5 y$.
(b) Solve this problem, making your method clear. Give, as fractions, the value of $C$ and the amount of each type of food that should be produced.

Another objective (for the same constraints given above) is to maximise $P=3 x+2 y$, where the variables must take integer values.
(c) Solve this problem, making your method clear. State the value of $P$ and the amount of each type of food that should be produced
8.

Figure 4


A trainee at a building company is using critical path analysis to help plan a project. Figure 4 shows the trainee's activity network. Each activity is represented by an arc and the number in brackets on each arc is the duration of the activity, in hours.
(a) Find the values of $x, y$ and $z$.
(b) State the total length of the project and list the critical activities.
(c) Calculate the total float time on
(i) activity $N$,
(ii) activity $H$.

The trainee's activity network is checked by the supervisor who finds a number of errors and omissions in the diagram. The project should be represented by the following precedence table

| Activity | Must be preceded by: | Duration |
| :---: | :---: | :---: |
| $A$ | - | 4 |
| $B$ | - | 3 |
| $C$ | - | 5 |
| $D$ | $B$ | 2 |
| $E$ | $A, D$ | 8 |
| $F$ | $B$ | 2 |
| $G$ | $C$ | 2 |
| $H$ | $C$ | 3 |
| $I$ | $F, G$ | 4 |
| $J$ | $F, G$ | 2 |
| $K$ | $H, J$ | 7 |
| $L$ | $E, I, K, M$ | 9 |
| $M$ | $E, I$ | 3 |
| $N$ | $H, J$ | 3 |
| $P$ | $Q$ | 6 |
| $Q$ |  | 5 |
| $R$ |  | 7 |

(d) By adding activities and dummies amend the diagram in the answer book so that it represents the precedence table. (The durations of activities $A, B, \ldots, N$ are all correctly given on the diagram in the answer book.)
(e) Find the total time needed to complete this project.

## Edexcel GCE

## Decision Mathematics D1

Advanced/Advanced Subsidiary
Tuesday 16 June 2004 - Afternoon
Time: $\mathbf{1}$ hour 30 minutes

Materials required for examination
Nil

Items included with question papers D1 Answer booklet

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates must NOT use algebra, differentiation and/or integration. Thus candidates must NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

## Instructions to Candidates

In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature

Information for Candidates
Full marks may be obtained for answers to ALL questions
This paper has seven questions.
Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. The organiser of a sponsored walk wishes to allocate each of six volunteers, Alan, Geoff, Laura, Nicola, Philip and Sam to one of the checkpoints along the route. Two volunteers are needed at checkpoint 1 (the start) and one volunteer at each of checkpoint 2, 3, 4 and 5 (the finish). Each volunteer will be assigned to just one checkpoint. The table shows the checkpoints each volunteer is prepared to supervise.

| Name | Checkpoints |
| :--- | :--- |
| Alan | 1 or 3 |
| Geoff | 1 or 5 |
| Laura | 2,1 or 4 |
| Nicola | 5 |
| Philip | 2 or 5 |
| Sam | 2 |

Initially Alan, Geoff, Laura and Nicola are assigned to the first checkpoint in their individual list.
(a) Draw a bipartite graph to model this situation and indicate the initial matching in a distinctive way.
(b) Starting from this initial matching, use the maximum matching algorithm to find an improved matching. Clearly list any alternating paths you use.
(c) Explain why it is not possible to find a complete matching.


Figure 1 shows a network of roads. The number on each edge gives the time, in minutes, to travel along that road. Avinash wishes to travel from $S$ to $T$ as quickly as possible.
(a) Use Dijkstra's algorithm to find the shortest time to travel from $S$ to $T$.
(b) Find a route for Avinash to travel from $S$ to $T$ in the shortest time. State, with a reason, whether this route is a unique solution

On a particular day Avinash must include $C$ in his route
(c) Find a route of minimal time from $S$ to $T$ that includes $C$, and state its time.

Figure 2

(a) Describe a practical problem that could be modelled using the network in Fig. 2 and solved using the route inspection algorithm.
(b) Use the route inspection algorithm to find which paths, if any, need to be traversed twice.
(c) State whether your answer to part (b) is unique. Give a reason for your answer.
(d) Find the length of the shortest inspection route that traverses each arc at least once and starts and finishes at the same vertex.

Given that it is permitted to start and finish the inspection at two distinct vertices,
(e) find which two vertices should be chosen to minimise the length of the route Give a reason for your answer.
4.

1. Glasgow
2. Newcastle
3. Manchester
4. York
5. Leicester
6. Birmingham
7. Cardiff
8. Exeter
9. Southampton
10. Plymouth

A binary search is to be performed on the names in the list above to locate the name Newcastle.
(a) Explain why a binary search cannot be performed with the list in its present form.
(b) Using an appropriate algorithm, alter the list so that a binary search can be performed. State the name of the algorithm you use.
(4)
(c) Use the binary search algorithm on your new list to locate the name Newcastle.

TURN OVER FOR QUESTION 5
5.

Figure 3


Figure 3 shows a capacitated directed network. The number on each arc is its capacity


Figure 4 shows a feasible initial flow through the same network.
(a) Write down the values of the flow $x$ and the flow $y$.
(b) Obtain the value of the initial flow through the network, and explain how you know it is not maximal.
(c) Use this initial flow and the labelling procedure on Diagram 1 in this answer book to find a maximum flow through the network. You must list each flow-augmenting route you use, together with its flow.
(d) Show your maximal flow pattern on Diagram 2.
(e) Prove that your flow is maximal.
6. The Young Enterprise Company "Decide", is going to produce badges to sell to decision maths students. It will produce two types of badges.

Badge 1 reads "I made the decision to do maths" and
Badge 2 reads "Maths is the right decision".
"Decide" must produce at least 200 badges and has enough material for 500 badges.
Market research suggests that the number produced of Badge 1 should be between $20 \%$ and $40 \%$ of the total number of badges made.

The company makes a profit of 30 p on each Badge 1 sold and 40 p on each Badge 2. It will sell all that it produced, and wishes to maximise its profit.

Let $x$ be the number produced of Badge 1 and $y$ be the number of Badge 2 .
(a) Formulate this situation as a linear programming problem, simplifying your inequalities so that all the coefficients are integers
(b) On the grid provided in the answer book, construct and clearly label the feasible region.
(c) Using your graph, advise the company on the number of each badge it should produce. State the maximum profit "Decide" will make.

TURN OVER FOR QUESTION 7

Figure 5


A project is modelled by the activity network shown in Fig. 5. The activities are represented by
the arcs. The number in brackets on each arc gives the time, in hours, to complete the activity. The numbers in circles give the event numbers. Each activity requires one worker
(a) Explain the purpose of the dotted line from event 4 to event 5 .
(b) Calculate the early time and the late time for each event. Write these in the boxes in the answer book
(c) Determine the critical activities
(d) Obtain the total float for each of the non-critical activities
(e) On the grid in the answer book, draw a cascade (Gantt) chart, showing the answers to parts (c) and (d).
(f) Determine the minimum number of workers needed to complete the project in the minimum time. Make your reasoning clear.

## 6689 <br> Edexcel GCE

## Decision Mathematics D1

Advanced/Advanced Subsidiary
Tuesday 2 November 2004 - Afternoon
Time: 1 hour 30 minutes

## Materials required for examination Nil <br> Items included with question papers D1 Answer booklet

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates must NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G

## Instructions to Candidates

Write your answers for this paper in the D1 answer book provided
In the boxes on the answer book, write your centre number, candidate number, your surname initial(s) and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy

## Information for Candidate

ull marks may be obtained for answers to ALL questions
This paper has eight questions.
The total mark for this paper is 75 . Page 12 is blank.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

## Write your answers for this paper in the D1 answer book.

1. 



Figure 1 shows a directed, capacitated network where the number on each arc is its capacity. A possible flow is shown from $S$ to $T$ and the value in brackets on each arc is the flow in that arc.
(a) Find the values of $x, y$ and $z$.
(3)
(b) Find, by inspection, the maximal flow from $S$ to $T$ and verify that it is maximal.
2. (a) Define the following terms
(i) planar graph,
(ii) Hamiltonian cycle.
(b) (i) Draw a graph of $\mathrm{K}_{3,2}$ in such a way as to show that it is planar.
(ii) Explain why the planarity algorithm cannot be used when drawing $K_{3,2}$ as a planar graph.
3. Six newspaper reporters Asif (A), Becky (B), Chris (C), David (D), Emma (E) and Fred (F), are to be assigned to six news stories Business (1), Crime (2), Financial (3), Foreign (4), Local (5) and Sport (6). The table shows possible allocations of reporters to news stories. For example, Chris can be assigned to any one of stories 1, 2 or 4.

|  | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A |  |  |  |  | $\checkmark$ |  |
| B | $\checkmark$ |  |  | $\checkmark$ |  |  |
| C | $\checkmark$ | $\checkmark$ |  | $\checkmark$ |  |  |
| D |  |  |  |  | $\checkmark$ |  |
| E |  |  | $\checkmark$ |  | $\checkmark$ | $\checkmark$ |
| F |  |  |  | $\checkmark$ |  |  |

(a) Show these possible allocations on the bipartite graph on the diagram in the answer book.
(1)

A possible matching i

$$
\text { A to } 5, \quad \text { C to } 1, \quad \text { E to } 6, \quad \text { F to } 4
$$

(b) Show this information, in a distinctive way, on the diagram in the answer book.
(c) Use an appropriate algorithm to find a maximal matching. You should list any alternating paths you have used.
(d) Explain why it is not possible to find a complete matching.
4.

45, 56,
37. 79,

46
18,
90,
81, 51
(a) Using the quick sort algorithm, perform one complete iteration towards sorting these numbers into ascending order.
(b) Using the bubble sort algorithm, perform one complete pass towards sorting the original list into descending order.
(2)

Another list of numbers, in ascending order, is

$$
7, \quad 23, \quad 31, \quad 37, \quad 41, \quad 44, \quad 50, \quad 62, \quad 71, \quad 73, \quad 94
$$

(c) Use the binary search algorithm to locate the number 73 in this list.


Figure 2 shows a network of roads connecting villages. The length of each road, in km, is shown. Village $B$ has only a small footbridge over the river which runs through the village. It can be accessed by two roads, from $A$ and $D$.

The driver of a snowplough, based at $F$, is planning a route to enable her to clear all the roads of snow. The route should be of minimum length. Each road can be cleared by driving along it once. The snowplough cannot cross the footbridge.

Showing all your working and using an appropriate algorithm,
(a) find the route the driver should follow, starting and ending at $F$, to clear all the roads of snow. Give the length of this route.

The local authority decides to build a road bridge over the river at $B$. The snowplough will be able to cross the road bridge.
(b) Reapply the algorithm to find the minimum distance the snowplough will have to travel (ignore the length of the new bridge).
6.

Figure 3


Peter wishes to minimise the time spent driving from his home $H$, to a campsite at $G$. Figure 3 shows a number of towns and the time, in minutes, taken to drive between them. The volume of traffic on the roads into $G$ is variable, and so the length of time taken to drive along these roads is expressed in terms of $x$, where $x \geq 0$.
(a) On the diagram in the answer book, use Dijkstra's algorithm to find two routes from $H$ to $G$ (one via $A$ and one via $B$ ) that minimise the travelling time from $H$ to $G$. State the length of each route in terms of $x$.
(b) Find the range of values of $x$ for which Peter should follow the route via $A$.
7.

Figure 4


The company EXYCEL makes two types of battery, X and Y. Machinery, workforce and predicted sales determine the number of batteries EXYCEL make. The company decides to use a graphical method to find its optimal daily production of X and Y

The constraints are modelled in Figure 4 where
$x=$ the number (in thousands) of type X batteries produced each day,
$y=$ the number (in thousands) of type Y batteries produced each day.
The profit on each type X battery is 40 p and on each type Y battery is 20 p . The company wishes to maximise its daily profit.
(a) Write this as a linear programming problem, in terms of $x$ and $y$, stating the objective function and all the constraints.
(b) Find the optimal number of batteries to be made each day Show your method cleat
(c) Find the daily profit, in $£$, made by EXYCEL.


| KEY |  |
| :--- | :--- |
| Early <br> event <br> time | Late <br> event <br> time |

The network in Figure 5 shows activities that need to be undertaken in order to complete a project. Each activity is represented by an arc. The number in brackets is the duration of the activity in hours. The early and late event times are shown at each node. The project can be completed in 24 hours
(a) Find the values of $x, y$ and $z$.
(b) Explain the use of the dummy activity in Figure 5
(c) List the critical activities
(d) Explain what effect a delay of one hour to activity $B$ would have on the time taken to complete the whole project.

The company which is to undertake this project has only two full time workers available. The project must be completed in 24 hours and in order to achieve this, the company is prepared to hire additional workers at a cost of $£ 28$ per hour. The company wishes to minimise the money spent on additional workers. Any worker can undertake any task and each task requires only one worker.
(e) Explain why the company will have to hire additional workers in order to complete the project in 24 hours.
(f) Schedule the tasks to workers so that the project is completed in 24 hours and at minimum cost to the company.
g) State the minimum extra cost to the company
(Total 17 marks)

## TOTAL FOR PAPER: 75 MARKS

END

## Edexcel GCE

## Decision Mathematics D1

## Advanced Subsidiary

Tuesday 18 January 2005 - Afternoon
Time: $\mathbf{1}$ hour 30 minutes
$\frac{\text { Materials required for examination }}{\text { Nil }} \quad \frac{\text { Items included with question papers }}{\text { D1 answer book }}$

㲘 algebra, differentiation and/or integration. Thus candidates may NOT use calculators HP 48G.

## Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), you entre number, candidate number, the unit title (Decision Mathematics D1), the paper reference (6689), your surname, other name and signature

The total mark for this paper is 75

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

## Write your answers in the D1 answer book for this paper.

1. 

Figure 1


The bipartite graph in Figure 1 shows a mapping between six people, Andy (A), David (D), Joan $(J)$, Preety $(P)$, Sally $(S)$ and Trevor $(T)$, and six tasks $1,2,3,4,5$ and 6.

The initial matching is $A$ to $2, D$ to $1, J$ to 3 and $P$ to 4
(a) Indicate this initial matching in a distinctive way on the bipartite graph drawn in the answer book.
(b) Starting from this initial matching, use the maximum matching algorithm to find a complete matching. List clearly the alternating paths you use.
2. The precedence table for activities involved in producing a computer game is shown below.

| Activity | Must be preceded by |
| :---: | :---: |
| $A$ | - |
| $B$ | - |
| $C$ | $B$ |
| $D$ | $A, C$ |
| $E$ | $A$ |
| $F$ | $E$ |
| $G$ | $E$ |
| $H$ | $G$ |
| $I$ | $G, F$ |
| $J$ | $G, I$ |
| $K$ | $H, K$ |
| $L$ |  |

An activity on arc network is to be drawn to model this production process.
(a) Explain why it is necessary to use at least two dummies when drawing the activity network.
(b) Draw the activity network using exactly two dummies.
3.

## Figure 2



The network in Figure 2 shows the distances, in metres, between 10 wildlife observation points. The observation points are to be linked by footpaths, to form a network along the arcs indicated, using the least possible total length.
(a) Find a minimum spanning tree for the network in Figure 2, showing clearly the order in which you selected the arcs for your tree, using
(i) Kruskal's algorithm,
(ii) Prim's algorithm, starting from $A$.

Given that footpaths are already in place along $A B$ and FI and so should be included in the spanning tree,
(b) explain which algorithm you would choose to complete the tree, and how it should be adapted. (You do not need to find the tree.)
4.

## $\begin{array}{llllllllll}650 & 431 & 245 & 643 & 455 & 134 & 710 & 234 & 162 & 452\end{array}$

(a) The list of numbers above is to be sorted into descending order. Perform a Quick Sort to obtain the sorted list, giving the state of the list after each pass, indicating the pivot elements

The numbers in the list represent the lengths, in mm, of some pieces of wood. The wood is sold in one metre lengths.
(b) Use the first-fit decreasing bin packing algorithm to determine how these pieces could be cut from the minimum number of one metre lengths. (You should ignore wastage due to cutting.)
(c) Determine whether your solution to part (b) is optimal. Give a reason for your answer


Figure 3 shows a network of paths. The number on each arc gives the distance, in metres, of that path.
(i) Use Dijkstra's algorithm to find the shortest distance from A to H
(ii) Solve the route inspection problem for the network shown in Figure 3. You should make your method and working clear. State a shortest route, starting at $A$, and find its length.
[The total weight of the network is 1241]
6.

Figure 4


Figure 4 shows a capacitated directed network. The number on each arc is its capacity.
(a) State the maximum flow along
(i) $S A D T$,
(ii) SCET,
(iii) $\operatorname{SBFT}$.
(b) Show these maximum flows on Diagram 1 in the answer book.

Take your answer to part $(b)$ as the initial flow pattern.
(c) (i) Use the labelling procedure to find a maximum flow from $S$ to $T$. Your working should be shown on Diagram 2 in the answer book. List each flow-augmenting route you use, together with its flow.
(ii) Draw your final flow pattern on Diagram 3 in the answer book.
(iii) Prove that your flow is maximal.
(d) Give an example of a practical situation that could have been modelled by the original network.
7. Flatland UK Ltd makes three types of carpet, the Lincoln, the Norfolk and the Suffolk. The carpets all require units of black, green and red wool

For each roll of carpet,
the Lincoln requires 1 unit of black, 1 of green and 3 of red,
the Norfolk requires 1 unit of black, 2 of green and 2 of red,
and the Suffolk requires 2 units of black, 1 of green and 1 of red
There are up to 30 units of black, 40 units of green and 50 units of red available each day. Profits of $£ 50, £ 80$ and $£ 60$ are made on each roll of Lincoln, Norfolk and Suffolk respectively. Flatland UK Ltd wishes to maximise its profit.

Let the number of rolls of the Lincoln, Norfolk and Suffolk made daily be $x, y$ and $z$ respectively.
(a) Formulate the above situation as a linear programming problem, listing clearly the constraints as inequalities in their simplest form, and stating the objective function.

This problem is to be solved using the Simplex algorithm. The most negative number in the profit row is taken to indicate the pivot column at each stage.
(b) Stating your row operations, show that after one complete iteration the tableau becomes

| Basic variable | $x$ | $y$ | $z$ | $r$ | $s$ | $t$ | Value |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $r$ | $\frac{1}{2}$ | 0 | $1 \frac{1}{2}$ | 1 | $-\frac{1}{2}$ | 0 | 10 |
| $y$ | $\frac{1}{2}$ | 1 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | 20 |
| $t$ | 2 | 0 | 0 | 0 | -1 | 1 | 10 |
| $P$ | -10 | 0 | -20 | 0 | 40 | 0 | 1600 |

(c) Explain the practical meaning of the value 10 in the top row.
(d) (i) Perform one further complete iteration of the Simplex algorithm.
(ii) State whether your current answer to part (d)(i) is optimal. Give a reason for your answer.
(iii) Interpret your current tableau, giving the value of each variable.
your answers in the D1 answer booklet for this paper.

## 6689/01

Edexcel GCE
Decision Mathematics D1

## Advanced/Advanced Subsidiary

Thursday 16 June 2005 - Afternoon
Time: $\mathbf{1}$ hour $\mathbf{3 0}$ minutes

## $\frac{\text { Materials required for examination }}{\text { Nil }} \quad \frac{\text { Items included with question papers }}{\text { D1 Answer book }}$

Candidates may use any calculator EXCEPT those with the facility for symboli algebra, differentiation and/or integration. Thus candidates must NOT us calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlet Packard HP 48G.

## Instructions to Candidates

Write your answers for this paper in the D1 answer book provided
In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature
When a calculator is used, the answer should be given to an appropriate degree of accuracy
Information for Candidate
Full marks may be obtained for answers to ALL questions
The marks for individual questions and the parts of questions are shown in round brackets:
e.g. (2)

This paper has eight questions. The total mark for this question paper is 75 .
Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit. order. Carry out a quick sort to produce a list of students in descending order of marks. You should show the result of each pass and identify your pivots clearly.
(Total 5 marks)
2.

| Ali | 74 |
| :---: | :---: |
| Bobby | 28 |
| Eun-Jung | 63 |
| Katie | 54 |
| Marciana | 54 |
| Peter | 49 |
| Rory | 37 |
| Sophie | 68 |

Figure 1

(a) Starting from $A$; write down a Hamiltonian cycle for the graph in Figure 1.
(b) Use the planarity algorithm to show that the graph in Figure 1 is planar.

Arcs $A F$ and $E F$ are now added to the graph.
(c) Explain why the new graph is not planar.
3.

## Figure 2



Figure 2 models a network of roads which need to be inspected to assess if they need to be resurfaced. The number on each arc represents the length, in km, of that road.

Each road must be traversed at least once and the length of the inspection route must be minimised.
(a) Starting and finishing at $A$, solve this route inspection problem. You should make your method and working clear. State the length of the shortest route
(The weight of the network is 77 km .)

Given that it is now permitted to start and finish the inspection at two distinct vertices,
(b) state which two vertices you should choose to minimise the length of the route. Give a reason for your answer.
4. The precedence table shows the activities involved in a project.

| Activity | Immediately preceding activities |
| :---: | :---: |
| $A$ | - |
| $B$ | - |
| $C$ | - |
| $D$ | $A$ |
| $E$ | $A$ |
| $F$ | $B$ |
| $G$ | $B$ |
| $H$ | $C, D$ |
| $I$ | $E, H$ |
| $J$ | $G, J$ |
| $K$ | $G$ |
| $L$ | $L$ |
| $M$ | $L$ |
| $N$ |  |

(a) Draw the activity network for this project, using activity on arc and using two dummies.
(b) Explain why each of the two dummies is necessary.
5.

Figure 3


Figure 4



E•

A film critic, Verity, must see five films A, B, C, D and E over two days.
The films are being shown at five special critics' preview times:
1 (Monday 4 pm ),
2 (Monday 7 pm ),
3 (Tuesday 1 pm ),
4 (Tuesday 4 pm ),
5 (Tuesday 7 pm ).
The bipartite graph in Figure 3 shows the times at which each film is showing. Initially Verity intends to see

Film A on Monday at 4 pm ,
Film B on Tuesday at 4 pm , Film C on Tuesday at 1 pm , Film D on Monday at 7 pm .

This initial matching is shown in Figure 4.
Using the maximum matching algorithm and the given initial matching,
(a) find two distinct alternating paths and complete the matchings they give.

Verity's son is very keen to see film D, but he can only go with his mother to the showing on Monday at 7 pm .
(b) Explain why it will not be possible for Verity to take her son to this showing and still see all five films herself.
6.

## Figure 5



Figure 5 shows a network of roads. The number on each arc represents the length of that road in km
(a) Use Dijkstra's algorithm to find the shortest route from $A$ to $J$. State your shortest route and its length.
(b) Explain how you determined the shortest route from your labelled diagram.

The road from $C$ to $F$ will be closed next week for repairs
(c) Find the shortest route from $A$ to $J$ that does not include $C F$ and state its length.
7. Polly has a bird food stall at the local market. Each week she makes and sells three types of pack $A, B$ and $C$.

Pack $A$ contains 4 kg of bird seed, 2 suet blocks and 1 kg of peanuts. Pack $B$ contains 5 kg of bird seed, 1 suet block and 2 kg of peanuts Pack $C$ contains 10 kg of bird seed, 4 suet blocks and 3 kg of peanuts

Each week Polly has 140 kg of bird seed, 60 suet blocks and 60 kg of peanuts available for the packs.

The profit made on each pack of $A, B$ and $C$ sold is $£ 3.50, £ 3.50$ and $£ 6.50$ respectively. Polly sells every pack on her stall and wishes to maximise her profit, $P$ pence.
Let $x, y$ and $z$ be the numbers of packs $A, B$ and $C$ sold each week.
An initial Simplex tableau for the above situation is

| Basic variable | $x$ | $y$ | $z$ | $r$ | $s$ | $t$ | Value |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $r$ | 4 | 5 | 10 | 1 | 0 | 0 | 140 |
| $s$ | 2 | 1 | 4 | 0 | 1 | 0 | 60 |
| $t$ | 1 | 2 | 3 | 0 | 0 | 1 | 60 |
| $P$ | -350 | -350 | -650 | 0 | 0 | 0 | 0 |

(a) Explain the meaning of the variables $r, s$ and $t$ in the context of this question.
(b) Perform one complete iteration of the Simplex algorithm to form a new tableau T. Take the most negative number in the profit row to indicate the pivotal column.
(c) State the value of every variable as given by tableau $T$.
(d) Write down the profit equation given by tableau $T$.
(e) Use your profit equation to explain why tableau $T$ is not optimal.

Taking the most negative number in the profit row to indicate the pivotal column,
(f) identify clearly the location of the next pivotal element
8.


Figure 6 shows a capacitated directed network. The number on each arc is its capacity. The numbers in circles show a feasible flow through the network. Take this as the initial flow.
(a) On Diagram 1 and Diagram 2 in the answer book, add a supersource $S$ and a supersink $T$. On Diagram 1 show the minimum capacities of the arcs you have added.

Diagram 2 in the answer book shows the first stage of the labelling procedure for the given initial flow.
(b) Complete the initial labelling procedure in Diagram 2.
(c) Find the maximum flow through the network. You must list each flow-augmenting route you use, together with its flow, and state the maximal flow.
(d) Show a maximal flow pattern on Diagram 3.
(e) Prove that your flow is maximal.
(f) Describe briefly a situation for which this network could be a suitable model.

Write your answers in the D1 answer booklet for this paper.

## 6689/01

## Edexcel GCE

Decision Mathematics D1

## Advanced/Advanced Subsidiary

Wednesday 18 January 2006 - Afternoon

## Time: $\mathbf{1}$ hour $\mathbf{3 0}$ minutes

## Materials required for examination Items included with question papers

Cadidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates must NOT use Hewlett Packard HP 48G

## Instructions to Candidates

Write your answers for this paper in the D1 answer book provided.
In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidate

Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2) There are 6 questions in this question paper. The total mark for this question paper is 75 .

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.
1.


Figure 2


A taxi firm has six taxis $A, B, C, D, E$ and $F$, available for six journeys, $1,2,3,4,5$ and 6 , which are booked for 9 a.m. tomorrow.

The bipartite graph shown in Figure 1 shows the possible matchings.
Initially $A, B, C$ and $D$ are matched to 5,2,3 and 6 respectively, as indicated in Figure 2.
(a) Explain why it is necessary to perform the maximum matching algorithm twice in order to try to obtain a complete matching.
b) Use the maximum matching algorithm twice to obtain a complete matching. List clearly the alternating paths you use.
2.

|  | $A$ | $B$ | $C$ | $D$ | $E$ | $F$ | $G$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $A$ | - | 48 | 117 | 92 | - | - | - |
| $B$ | 48 | - | - | - | - | 63 | 55 |
| $C$ | 117 | - | - | 28 | - | - | 85 |
| $D$ | 92 | - | 28 | - | 58 | 132 | - |
| $E$ | - | - | - | 58 | - | 124 | - |
| $F$ | - | 63 | - | 132 | 124 | - | - |
| $G$ | - | 55 | 85 | - | - | - | - |

The table shows the lengths, in metres, of the paths between seven vertices $A, B, C, D, E, F$ and $G$ in a network N .
(a) Use Prim's algorithm, starting at $A$, to solve the minimum connector problem for this table of distances. You must clearly state the order in which you selected the edges of your tree, and the weight of your final tree. Draw your tree using the vertices given in Diagram 1 in the answer book.
(b) Draw N using the vertices given in Diagram 2 in the answer book
(c) Solve the Route Inspection problem for N . You must make your method of working clear State a shortest route and find its length. (The weight of N is 802 .)
3.


An algorithm is described by the flow chart shown in Figure 3.
(a) Complete the table in the answer book recording the results of each step as the algorithm is applied.
(Notice that values of $A, B, C$ and $D$ are to be given to 3 decimal places, and the values of $E$ to 1 significant figure.)
(b) Write down the output from the algorithm
4. (a) Define the terms
(i) cut,
(ii) minimum cut,
as applied to a directed network flow.

## Figure 4



Figure 4 shows a capacitated directed network and two cuts $C_{1}$ and $C_{2}$. The number on each arc is its capacity.
(b) State the values of the cuts $C_{1}$ and $C_{2}$

Given that one of these two cuts is a minimum cut,
(c) find a maximum flow pattern by inspection, and show it on the diagram in the answer book.
(d) Find a second minimum cut for this network.

In order to increase the flow through the network it is decided to add an arc of capacity 100 joining $D$ either to $E$ or to $G$.
(e) State, with a reason, which of these arcs should be added, and the value of the increased flow.
5.


The network in Figure 5 shows the activities involved in a process. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, taken to complete the activity.
(a) Calculate the early time and late time for each event, showing them on the diagram in the answer book.
(b) Determine the critical activities and the length of the critical path
(c) On the grid in the answer book, draw a cascade (Gantt) chart for the process.

Each activity requires only one worker, and workers may not share an activity.
(d) Use your cascade chart to determine the minimum numbers of workers required to complete the process in the minimum time. Explain your reasoning clearly
(e) Schedule the activities, using the number of workers you found in part (d), so that the process is completed in the shortest time.
6. A company produces two types of party bag, Infant and Junior. Both types of bag contain balloon, a toy and a whistle. In addition the Infant bag contains 3 sweets and 3 stickers and the Junior bag contains 10 sweets and 2 stickers.

The sweets and stickers are produced in the company's factory. The factory can produce up to 3000 sweets per hour and 1200 stickers per hour. The company buys a large supply of balloons, toys and whistles.

Market research indicates that at least twice as many Infant bags as Junior bags should be produced.

Both types of party bag are sold at a profit of 15 p per bag. All the bags are sold. The company wishes to maximise its profit

Let $x$ be the number of Infant bags produced and $y$ be the number of Junior bags produced per hour
(a) Formulate the above situation as a linear programming problem.
(b) Represent your inequalities graphically, indicating clearly the feasible region
(c) Find the number of Infant bags and Junior bags that should be produced each hour and the maximum hourly profit. Make your method clear.

In order to increase the profit further, the company decides to buy additional equipment. It can buy equipment to increase the production of either sweets or stickers, but not both.
(d) Using your graph, explain which equipment should be bought, giving your reasoning

The manager of the company does not understand why the balloons, toys and whistles have not been considered in the above calculations.
(e) Explain briefly why they do not need to be considered.

## 6689/01

## Edexcel GCE

Decision Mathematics D1

## Advanced/Advanced Subsidiary

## Wednesday 24 May 2006 - Afternoon

## Time: 1 hour 30 minutes

## $\frac{\text { Materials required for examination }}{\text { Nil }} \frac{\text { Items included with question papers }}{\text { D1 Answer book }}$

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates must NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

## Instructions to Candidate

Write your answers for this paper in the D1 answer book provided
In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy

## nformation for Candidates

Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2) There are 7 questions in this question paper. The total mark for this question paper is 75 .

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Write your answers in the D1 answer booklet for this paper.
1.

The list of numbers above is to be sorted into descending order. Perform a bubble sort to obtain the sorted list, giving the state of the list after each completed pass.
2. (a) Define the term 'alternating path'.
(2)


The bipartite graph in Figure 1 shows the films that six customers wish to hire this Saturday evening. The shop has only one copy of each film. The bold lines indicate an initial matching.
(b) Starting from this initial matching use the maximum matching algorithm twice to obtain a complete matching. You should clearly state the alternating paths you use.

Figure 2


Figure 2 shows the network of pipes represented by arcs. The length of each pipe, in kilometres, Figure 2 shows the network of pipes represented by arcs. The length of each pipe, in kilometres,
is shown by the number on each arc. The network is to be inspected for leakages, using the shortest route and starting and finishing at $A$.
(a) Use the route inspection algorithm to fins which arcs, if any, need to be traversed twice.
(b) State the length of the minimum route. [The total weight of the network is 394 km .]

It is now permitted to start and finish the inspection at two distinct vertices
(c) State, with a reason, which two vertices should be chosen to minimise the length of the new route.
4. (a) Explain what is meant by the term 'path'
(2)


Figure 3 shows a network of cycle tracks. The number on each edge represents the length, in miles, of that track. Mary wishes to cycle from $A$ to $I$ as part of a cycling holiday. She wishes to minimise the distance she travels.
(b) Use Dijkstra's algorithm to find the shortest path from A to $I$. Show all necessary working in the boxes in Diagram 1 in the answer book. State your shortest path and its length.
(c) Explain how you determined the shortest path from your labelling

Mary wants to visit a theme park at $E$.
(d) Find a path of minimal length that goes from $A$ to $I$ via $E$ and state its length.

## Figure 4



An engineering project is modelled by the activity network shown in Figure 4. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires one worker. The project is to be completed in the shortest time.
(a) Calculate the early time and late time for each event. Write these in boxes in Diagram 1 in the answer book.
(b) State the critical activities.
(c) Find the total float on activities $D$ and $F$. You must show your working.
(d) On the grid in the answer book, draw a cascade (Gantt) chart for this project.

The chief engineer visits the project on day 15 and day 25 to check the progress of the work. Given that the project is on schedule,
(e) which activities must be happening on each of these two days?
6. The tableau below is the initial tableau for a maximising linear programming problem.

| Basic variable | $x$ | $y$ | $z$ | $r$ | $s$ | $t$ | Value |
| :---: | :---: | ---: | ---: | :---: | :---: | :---: | :---: |
| $r$ | 7 | 10 | 10 | 1 | 0 | 0 | 3600 |
| $s$ | 6 | 9 | 12 | 0 | 1 | 0 | 3600 |
| $t$ | 2 | 3 | 4 | 0 | 0 | 1 | 2400 |
| $P$ | -35 | -55 | -60 | 0 | 0 | 0 | 0 |

(a) Write down the four equations represented in the initial tableau above.
(4)
(b) Taking the most negative number in the profit row to indicate the pivot column at each stage, solve this linear programming problem. State the row operations that you use.
(c) State the values of the objective function and each variable.

## Figure 5



Figure 5 shows a capacitated, directed network. The capacity of each arc is shown on each arc. The numbers in circles represent an initial flow from $S$ to $T$.

Two cuts $C_{1}$ and $C_{2}$ are shown on Figure 5
(a) Write down the capacity of each of the two cuts and the value of the initial flow.
b) Complete the initialisation of the labelling procedure on Diagram 1 by entering values along $\operatorname{arcs} A C, C D, D E$ and $D T$.
c) Hen each flow-augmenting path you use, together with its flow.
(d) Show your maximal flow pattern on Diagram 2.
(e) Prove that your flow is maximal.

## 6689/01 <br> Edexcel GCE

Decision Mathematics D1
Advanced/Advanced Subsidiary
Friday 12 January 2007 - Morning
Time: 1 hour 30 minutes

## Materials required for examination Items included with question papers D1 answer book

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates mus NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G

## Instructions to Candidates

Write your answers for this paper in the D1 answer book provided
In the boxes on the answer book, write your centre number, candidate number, your surname initial(s) and signature
When a calculator is used, the answer should be given to an appropriate degree of accuracy.
You must complete your answers in blue or black ink or pencil.
Information for Candidates
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2) here are 8 questions in this question paper.
The total mark for this paper is 75 . There are 12 pages in this question paper.
The answer book has 16 pages. Any blank pages are indicated.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.



1. Use the binary search algorithm to try to locate the name NIGEL in the following alphabetical list. Clearly indicate how you chose your pivots and which part of the list is being rejected at each stage

| Bhavika |  |
| :--- | :--- |
| Clive |  |
| Elizabeth |  |
| John |  |
| Mark |  |
| Nicky |  |
| 6. | Preety |
| 8. | Steve |
| Trevor |  |
| 1. | Verity |

2. 



Figure 1 shows the possible allocations of five people, Ellen, George, Jo, Lydia and Yi Wen to five tasks, 1, 2, 3, 4 and 5.

Figure 2 shows an initial matching.
(a) Find an alternating path linking George with 5. List the resulting improved matching this gives.
(b) Explain why it is not possible to find a complete matching.
3.

(a) Write down the name given to the type of graph drawn in Figure 3

A Hamiltonian cycle for the graph in Figure 3 begins A, 3, B, ...
(b) Complete this Hamiltonian cycle.
(c) Starting with the Hamiltonian cycle found in (b), use the planarity algorithm to determine if the graph is planar.

George now has task 2 added to his possible allocation.
(c) Using the improved matching found in part (a) as the new initial matching, find an alternating path linking Yi Wen with task 1 to find a complete matching. List the complete matching.
4. A three-variable linear programming problem in $x, y$ and $z$ is to be solved. The objective is to maximise the profit $P$. The following initial tableau was obtained.

| Basic variable | $x$ | $y$ | $z$ | $r$ | $s$ | Value |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $r$ | 2 | 0 | 4 | 1 | 0 | 80 |
| $s$ | 1 | 4 | 2 | 0 | 1 | 160 |
| $P$ | -2 | -8 | -20 | 0 | 0 | 0 |

(a) Taking the most negative number in the profit row to indicate the pivot column, perform one complete iteration of the simplex algorithm, to obtain tableau $T$. State the row operations that you use.
(b) Write down the profit equation shown in tableau $T$.
(c) State whether tableau $T$ is optimal. Give a reason for your answer.
5. (a) Explain why a network cannot have an odd number of vertices of odd degree


Figure 4 shows a network of paths in a public park. The number on each arc represents the length of that path in metres. Hamish needs to walk along each path at least once to check the paths fo frost damage starting and finishing at $A$. He wishes to minimise the total distance he walks.
(b) Use the route inspection algorithm to find which paths, if any, need to be traversed twice
(c) Find the length of Hamish's route.
[The total weight of the network in Figure 4 is 4180 m .]
6.

Figure 5


A project is modelled by the activity network shown in Figure 5. The activities are represented by the arcs. The number in brackets on each arc gives the time, in hours, to complete the activity. The numbers in circles are the event numbers. Each activity requires one worker.
(a) Explain the purpose of the dotted line from event 6 to event 8 .
(b) Calculate the early time and late time for each event. Write these in the boxes in the answer book.
(4)
(c) Calculate the total float on activities $D, E$ and $F$.
(d) Determine the critical activities
(e) Given that the sum of all the times of the activities is 95 hours, calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must show your working.
(f) Given that workers may not share an activity, schedule the activities so that the process is completed in the shortest time using the minimum number of workers.
7.

Figure 6


The captain of the Malde Mare takes passengers on trips across the lake in her boat.
The number of children is represented by $x$ and the number of adults by $y$.
Two of the constraints limiting the number of people she can take on each trip are

$$
\begin{array}{cc}
\text { and } & x<10 \\
& 2 \leqslant y \leqslant 10
\end{array}
$$

These are shown on the graph in Figure 6, where the rejected regions are shaded out.
(a) Explain why the line $x=10$ is shown as a dotted line.
(b) Use the constraints to write down statements that describe the number of children and the number of adults that can be taken on each trip.

For each trip she charges $£ 2$ per child and $£ 3$ per adult. She must take at least $£ 24$ per trip to cover costs.

The number of children must not exceed twice the number of adults.
(c) Use this information to write down two inequalities.
(d) Add two lines and shading to Diagram 1 in your answer book to represent these inequalities. Hence determine the feasible region and label it R.
(e) Use your graph to determine how many children and adults would be on the trip if the captain takes:
(i) the minimum number of passengers,
(ii) the maximum number of passengers.
8.

In solving a network flow problem using the labelling procedure, the diagram in Figure 7 was created.
The arrow on each arc indicates the direction of the flow along that arc.
The arrows above and below each arc show the direction and value of the flow as indicated by the labelling procedure.
(a) Add a supersource S , a supersink T and appropriate arcs to Diagram 2 in the answer book, and complete the labelling procedure for these arcs.
(b) Write down the value of the initial flow shown in Figure 7.
(c) Use Diagram 2, the initial flow and the labelling procedure to find the maximal flow of 124 through this network. List each flow-augmenting path you use, together with its flow.
(d) Show your flow on Diagram 3 and state its value.
(e) Prove that your flow is maximal.

## 6689/01

## Edexcel GCE

Decision Mathematics D1
Advanced/Advanced Subsidiary
Tuesday 5 June 2007 - Afternoon
Time: 1 hour 30 minutes
$\begin{array}{lll}\frac{\text { Materials required for examination }}{\text { Nil }} & & \text { Items included with question papers } \\ \text { D1 Answer book } \\ \text { Candidates may use any calculator allowed by the regulations of the Joint }\end{array}$
Candidates may use any calculator allowed by the regulations of the
Council for Qualifications. Calculators must not have the facility for Council for Qualifications. Calculators must not have the facility for symboitc algebra manipulation, diferentiation and

## Instructions to Candidates

Write your answers for this paper in the D1 answer book provided.
In the boxes on the answer book, write your centre number, candidate number, your surname initial(s) and signature
When a calculator is used, the answer should be given to an appropriate degree of accuracy
Complete your answers in blue or black ink or pencil.
Do not return the question paper with the answer book.
Information for Candidates
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2) There are 8 questions in this question paper. The total mark for this paper is 75
There are 12 pages in this question paper. The answer book has 16 pages. Any blank pages are indicated

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner
Answers without working may not gain full credit.

1. Explain what is meant by a planar graph.
2. 



Figure 1

Six workers, Annie, Emma, Hannah, Jerry, Louis and Morand, are to be assigned to five tasks, $1,2,3,4$ and 5 .

For safety reasons, task 1 must be done by two people working together.
A bipartite graph showing the possible allocations of the workers is given in Figure 1 and an initial matching is given in Figure 2.

The maximum matching algorithm will be used to obtain a complete matching.
(a) Although there are five tasks, six vertices have been created on the right hand side of each bipartite graph. Explain why this is necessary when applying this algorithm.
(b) Find an alternating path and the complete matching it gives.

Hannah is now unable to do task 5 due to health reasons.
(c) Explain why a complete matching is no longer possible.
3.


Figure 3
An algorithm is described by the flow chart shown in Figure 3.
(a) Given that $x=54$ and $y=63$, complete the table in the answer book to show the results obtained at each step when the algorithm is applied.
(b) State what the algorithm achieves.
4.


Figure 4 models a network of underground tunnels that have to be inspected. The number on each arc represents the length, in km , of each tunnel.

Joe must travel along each tunnel at least once and the length of his inspection route must be minimised.

The total weight of the network is 125 km
The inspection route must start and finish at A.
(a) Use an appropriate algorithm to find the length of the shortest inspection route. You should make your method and working clear.

Given that it is now permitted to start and finish the inspection at two distinct vertices,
(b) state which two vertices should be chosen to minimise the length of the new route. Give a reason for your answer.
5.

|  | $M$ | $A$ | $B$ | $C$ | $D$ | $E$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $M$ | - | 215 | 170 | 290 | 210 | 305 |
| $A$ | 215 | - | 275 | 100 | 217 | 214 |
| $B$ | 170 | 275 | - | 267 | 230 | 200 |
| $C$ | 290 | 100 | 267 | - | 180 | 220 |
| $D$ | 210 | 217 | 230 | 180 | - | 245 |
| $E$ | 305 | 214 | 200 | 220 | 245 | - |

The table shows the cost, in pounds, of linking five automatic alarm sensors, $A, B, C, D$ and $E$, and the main reception, $M$.
(a) Use Prim's algorithm, starting from $M$, to find a minimum spanning tree for this table of costs. You must list the arcs that form your tree in the order that they are selected.
(b) Draw your tree using the vertices given in Diagram 1 in the answer book.
(c) Find the total weight of your tree.
(d) Explain why it is not necessary to check for cycles when using Prim's algorithm.
6.


Figure 5
The network in Figure 5 shows the activities that need to be undertaken to complete a project. Each ctivity is represented by an arc. The number in brackets is the duration of the activity in days. The early and late event times are to be shown at each vertex and some have been completed for you.
(a) Calculate the missing early and late times and hence complete Diagram 2 in your answer book.
(b) List the two critical paths for this network.
(c) Explain what is meant by a critical path.

The sum of all the activity times is 110 days and each activity requires just one worker. The project must be completed in the minimum time.
(d) Calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must show your working.
(e) List the activities that must be happening on day 20 .
(f) Comment on your answer to part (e) with regard to the lower bound you found in part (d).
(g) Schedule the activities, using the minimum number of workers, so that the project is completed in 30 days.


## Figure 6

Figure 6 shows a capacitated, directed network. The number on each arc represents the capacity of that arc. The numbers in circles represent an initial flow.
(a) State the value of the initial flow.
(b) State the capacities of cuts $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$.

Diagram 3 in the answer book shows the labelling procedure applied to the above network.
(c) Using Diagram 3, increase the flow by a further 19 units. You must list each flow-augmenting path you use, together with its flow.
(d) Prove that the flow is now maximal.

## 6689/01 <br> Edexcel GCE

## Decision Mathematics D1 <br> Advanced/Advanced Subsidiary

Friday 11 January 2008 - Morning
Time: 1 hour 30 minutes
Materials required for examination $\quad$ Items included with question papers
D1 Answer book
symbolic algebra manipulation, differentiation and integration, for retrievable mathematical formulae stored in them.

## nstructions to Candidates

Write your answers for this paper in the D1 answer book provided.
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When a calculator is used, the answer should be given to an appropriate degree of accuracy. Complete your answers in blue or black ink or pencil.
Do not return the question paper with the answer book
Information for Candidates
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2). There are 7 questions in this question paper. The total mark for this paper is 75 .
There are 12 pages in this question paper. The answer book has 16 pages. Any blank pages are indicated.

Advice to Candidates
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You should show sufficient working to make your methods clear to the Examiner
Answers without working may not gain full credit.

N29279A

1. (a) Define the terms
(i) alternating path
(ii) matching.
(2)


Figure


Figure 2

At a school fair, five teachers, $A, B, C, D$ and $E$, are to supervise five stalls, 1, 2, 3, 4 and 5
A bipartite graph showing their possible allocations is given in Figure 1. An initial matching is given in Figure 2.
(b) Use the maximum matching algorithm twice to obtain a complete matching. List clearly the alternating paths you use.
2. (a)

18
20
11
7
17
15
14
21
23
16
The list of numbers shown above is to be sorted into ascending order. Apply quick sort to obtain the sorted list. You must make your pivots clear.


## Figure 3

Figure 3 represents a network of paths in a park. The number on each arc represents the length of the path in metres
(b) Using your answer to part (a) and Kruskal's algorithm, find a minimum spanning tree for the network in Figure 3. You should list the arcs in the order in which you consider them and state whether you are adding it to your minimum spanning tree.
(c) Find the total weight of the minimum spanning tree
3.


Figure 4

Figure 4 models a network of roads in a housing estate. The number on each arc represents the length, in km, of the road.

The total weight of the network is 11 km
A council worker needs to travel along each road once to inspect the road surface. He will start and finish at A and wishes to minimise the length of his route.
(a) Use an appropriate algorithm to find a route for the council worker. You should make your method and working clear. State your route and its length.

A postal worker needs to walk along each road twice, once on each side of the road. She must start and finish at A. The length of her route is to be minimised. You should ignore the width of the road.
(b) (i) Explain how this differs from the standard route inspection problem.
(ii) Find the length of the shortest route for the postal worker.
4.


Figure 5

A project is modelled by the activity network shown in Figure 5. The activities are represented by the arcs. The number in brackets on each arc gives the time, in hours, to complete the activity. Some of the early and late times for each event are shown.
(a) Calculate the missing early and late times and hence complete Diagram 1 in your answer book.
(b) Calculate the total float on activities D, G and I. You must make your calculations clear.
(c) List the critical activities

Each activity requires one worker.
(d) Calculate a lower bound for the number of workers needed to complete the project in the minimum time.
5. (a) Draw the activity network described in this precedence table, using activity on arc and exactly
two dummies.
(4)

| Activity | Immediately preceding activities |
| :---: | :---: |
| $A$ | - |
| $B$ | - |
| $C$ | $A$ |
| $D$ | $B$ |
| $E$ | $B, C$ |
| $F$ |  |

(b) Explain why each of the two dummies is necessary.
6. (a) Define the term 'cut' as it applies to a directed network.


Figure 6

Figure 6 shows a capacitated, directed network. The number on each arc represents the capacity of that arc. The numbers in circles represent an initial flow.
(b) Complete the labelling procedure on Diagram 2 in your answer book by entering values along CE, EG, HT and GT.
(c) Find the maximum flow through the network. You must list each flow-augmenting route you use together with its flow.
(d) Show a maximal flow pattern on Diagram 3 in your answer book.
(e) State the value of the maximum flow through the network.
(f) Prove that your flow is maximal.


Figure 7
7. Phil sells boxed lunches to travellers at railway stations. Customers can select either the vegetarian box or the non-vegetarian box

Phil decides to use graphical linear programming to help him optimise the numbers of each type of box he should produce each day.

Each day Phil produces $x$ vegetarian boxes and $y$ non-vegetarian boxes
One of the constraints limiting the number of boxes is

$$
x+y \geqslant 70 .
$$

This, together with $x \geqslant 0, y \geqslant 0$ and a fourth constraint, has been represented in Figure 7. The rejected region has been shaded
(a) Write down the inequality represented by the fourth constraint

Two further constraints are:

$$
\begin{aligned}
& x+2 y \leqslant 160 \\
& \text { and } \quad y>60 .
\end{aligned}
$$

(b) Add two lines and shading to Diagram 4 in your answer book to represent these inequalities.
(c) Hence determine and label the feasible region, R
(d) Use your graph to determine the minimum total number of boxes he needs to prepare each day. Make your method clear

Phil makes a profit of $£ 1.20$ on each vegetarian box and $£ 1.40$ on each non-vegetarian box. He wishes to maximise his profit
(e) Write down the objective function.
(f) Use your graph to obtain the optimal number of vegetarian and non-vegetarian boxes he should produce each day. You must make your method clear.
(g) Find Phil's maximum daily profit.

## 6689/01 <br> Edexcel GCE

## Decision Mathematics D1 <br> Advanced/Advanced Subsidiary

Thursday 15 May 2008 - Morning
Time: 1 hour 30 minutes

| Materials required for examination | Items included with question papers |
| :--- | :--- |
|  | D1 Answer book |

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Information for Candidates
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There are 8 pages in this question paper. The answer book has 16 pages. Any blank pages are indicated

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner
Answers without working may not gain full credit.
1.

| 29 | 52 | 73 | 87 | 74 | 47 | 38 | 61 | 41 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

The numbers in the list represent the lengths in minutes of nine radio programmes. They are to be recorded onto tapes which each store up to 100 minutes of programmes.
(a) Obtain a lower bound for the number of tapes needed to store the nine programmes
b) Use the first-fit bin packing algorithm to fit the programmes onto the tapes
(c) Use the first-fit decreasing bin packing algorithm to fit the programmes onto the tapes.


Five tour guides, Alice, Emily, George, Rose and Weidi, need to be assigned to five coach trips , 2, 3, 4 and 5. A bipartite graph showing their preferences is given in Figure 1 and an initial matching is given in Figure 2.
(a) Use the maximum matching algorithm, starting with vertex G, to increase the number of matchings. State the alternating path you used
b) List the improved matching you found in (a).
(c) Explain why a complete matching is not possible

Weidi agrees to be assigned to coach trip 3, 4 or 5 .
(d) Starting with your current maximal matching, use the maximum matching algorithm to obtain a complete matching.
3.


Figure 3
Figure 3 shows a network of roads. The number on each arc represents the length, in km, of that road.
(a) Use Dijkstra's algorithm to find the shortest route from A to I. State your shortest route and its length.

Sam has been asked to inspect the network and assess the condition of the roads. He must travel along each road at least once, starting and finishing at A.
(b) Use an appropriate algorithm to determine the length of the shortest route Sam can travel. State a shortest route.
(The total weight of the network is 197 km )
5.


Figure 5
Figure 5 shows a capacitated, directed network of pipes. The number on each arc represents the capacity of that pipe. The numbers in circles represent a feasible flow.
(a) State the values of $x$ and $y$.
(2)
(b) List the saturated arcs.
(c) State the value of the feasible flow.
(d) State the capacities of the cuts $\mathrm{C}_{1}, \mathrm{C}_{2}$, and $\mathrm{C}_{3}$.
(e) By inspection, find a flow-augmenting route to increase the flow by one unit. You must state your route
(f) Prove that the new flow is maximal.
6. The tableau below is the initial tableau for a maximising linear programming problem in $x, y$ and $z$.

| Basic variable | $x$ | $y$ | $z$ | $r$ | $s$ | $t$ | Value |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $r$ | 4 | $\frac{7}{3}$ | $\frac{5}{2}$ | 1 | 0 | 0 | 64 |
| $s$ | 1 | 3 | 0 | 0 | 1 | 0 | 16 |
| $t$ | 4 | 2 | 2 | 0 | 0 | 1 | 60 |
| $P$ | -5 | $\frac{-7}{2}$ | -4 | 0 | 0 | 0 | 0 |

(a) Taking the most negative number in the profit row to indicate the pivot column at each stage, perform two complete iterations of the simplex algorithm. State the row operations you use.
(b) Explain how you know that your solution is not optimal.
7.


## Figure 6

The network in Figure 6 shows the activities that need to be undertaken to complete a building project. Each activity is represented by an arc. The number in brackets is the duration of the activity in days. The early and late event times are shown at each vertex.
(a) Find the values of $v, w, x, y$ and $z$.
(b) List the critical activities.
(c) Calculate the total float on each of activities H and J
(d) Draw a cascade (Gantt) chart for the project

The engineer in charge of the project visits the site at midday on day 8 and sees that activity E has not yet been started
(e) Determine if the project can still be completed on time. You must explain your answer.

Given that each activity requires one worker and that the project must be completed in 35 days,
(f) use your cascade chart to determine a lower bound for the number of workers needed. You must justify your answer.
8. Class 8 B has decided to sell apples and bananas at morning break this week to raise money for charity. The profit on each apple is 20 p, the profit on each banana is 15 p. They have done some market research and formed the following constraints.

- They will sell at most 800 items of fruit during the week
- They will sell at least twice as many apples as bananas.
- They will sell between 50 and 100 bananas

Assuming they will sell all their fruit, formulate the above information as a linear programming problem, letting $a$ represent the number of apples they sell and $b$ represent the number of bananas they sell.

Write your constraints as inequalities

## 6689/01

## Edexcel GCE

Decision Mathematics D1
Advanced/Advanced Subsidiary
Tuesday 13 January 2009 - Morning
Time: 1 hour 30 minutes

> Materials required for examination $\quad$ Items included with question papers D1 Answer book Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

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There are 8 pages in this question paper. The answer book has 20 pages. Any blank pages are indicated

Advice to Candidates
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1.
Max Lauren John Hannah Kieran Tara Richard Imogen
(a) Use a quick sort to produce a list of these names in ascending alphabetical order. You must make your pivots clear
(b) Use the binary search algorithm on your list from part (a) to try to locate the name 'Hugo'.
2.

|  | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{D}$ | $\mathbf{E}$ | $\mathbf{F}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{A}$ | - | 24 | - | - | 23 | 22 |
| $\mathbf{B}$ | 24 | - | 18 | 19 | 17 | 20 |
| $\mathbf{C}$ | - | 18 | - | 11 | 14 | - |
| $\mathbf{D}$ | - | 19 | 11 | - | 13 | - |
| $\mathbf{E}$ | 23 | 17 | 14 | 13 | - | 21 |
| $\mathbf{F}$ | 22 | 20 | - | - | 21 | - |

The table shows the distances, in metres, between six vertices, $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{E}$ and $\mathbf{F}$, in a network.
(a) Draw the weighted network using the vertices given in Diagram 1 in the answer booklet.
(b) Use Kruskal's algorithm to find a minimum spanning tree. You should list the edges in the order that you consider them and state whether you are adding them to your minimum spanning tree
(c) Draw your tree on Diagram 2 in the answer booklet and find its total weight.
3. (a) Draw the activity network described in this precedence table, using activity on arc and exactly two dummies.

| Activity | Immediately preceding activities |
| :---: | :---: |
| $\mathbf{A}$ | - |
| B | - |
| $\mathbf{C}$ | - |
| D | B |
| E | B, $\mathbf{C}$ |
| F | B, $\mathbf{C}$ |
| $\mathbf{G}$ | F |
| H | F |
| I | G, $\mathbf{H}$ |
| J | I |

(b) Explain why each of the two dummies is necessary.
4.


Figure 1


Figure 2

Figure 1 shows the possible allocations of six people, A, B, C, D, E and F, to six tasks, 1, 2, 3, 4, 5 and 6.
Figure 2 shows an initial matching.
(a) Starting from this initial matching, use the maximum matching algorithm to find an improved matching. You must list the alternating path used, and your improved matching.
(b) Explain why it is not possible to find a complete matching.

D now has task 2 added to their possible allocation.
(c) Using the improved matching found in part (a) as the new initial matching, use the maximum matching algorithm to find a complete matching. You must list the alternating path used and your complete matching.
5.


Figure 3
(The total weight of the network in Figure 3 is 543 km .)
Figure 3 models a network of railway tracks that have to be inspected. The number on each arc is the length, in km, of that section of railway track.
Each track must be traversed at least once and the length of the inspection route must be minimised.
The inspection route must start and finish at the same vertex.
(a) Use an appropriate algorithm to find the length of the shortest inspection route. You should make your method and working clear.

It is now permitted to start and finish the inspection at two distinct vertices
(b) State which two vertices should be chosen to minimise the length of the new route. Give a reason for your answer.
7. A linear programming problem is modelled by the following constraints

$$
\begin{aligned}
8 x+3 y & \leqslant 480 \\
8 x+7 y & \geqslant 560 \\
y & \geqslant 4 x \\
x, y & \geqslant 0
\end{aligned}
$$

(a) Use the grid provided in your answer book to represent these inequalities graphically. Hence determine the feasible region and label it R .

The objective function, $F$, is given by

$$
F=3 x+y
$$

(b) Making your method clear, determine
(i) the minimum value of the function $F$ and the coordinates of the optimal point,
(ii) the maximum value of the function $F$ and the coordinates of the optimal point.
8.


## Figure 5

The network in Figure 5 shows the activities involved in a process. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, taken to complete the activity
(a) Calculate the early time and the late time for each event, showing them on the diagram in the answer book.
(b) Determine the critical activities and the length of the critical path.
(c) Calculate the total float on activities F and G. You must make the numbers you used in your calculation clear
(d) On the grid in the answer book, draw a cascade (Gantt) chart for the process.

Given that each task requires just one worker
(e) use your cascade chart to determine the minimum number of workers required to complete the process in the minimum time. Explain your reasoning clearly.

## TOTAL FOR PAPER: 75 MARKS

END

## 6689/01 <br> Edexcel GCE

Decision Mathematics D1
Advanced/Advanced Subsidiary
Wednesday 20 May 2009 - Afternoon
Time: 1 hour 30 minutes

> | Materials required for examination $\quad$ Items included with question papers |
| :--- |
| Nil |
| Candidates may use any calculator allowed by the regulations of the Joint |
| Council for Qualifications. Calculators must not have the facility for |
| $\begin{array}{l}\text { Cor }\end{array}$ | ymbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them

1. 

|  | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{D}$ | $\mathbf{E}$ | $\mathbf{F}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{A}$ | - | 135 | 180 | 70 | 95 | 225 |
| $\mathbf{B}$ | 135 | - | 215 | 125 | 205 | 240 |
| $\mathbf{C}$ | 180 | 215 | - | 150 | 165 | 155 |
| $\mathbf{D}$ | 70 | 125 | 150 | - | 100 | 195 |
| $\mathbf{E}$ | 95 | 205 | 165 | 100 | - | 215 |
| $\mathbf{F}$ | 225 | 240 | 155 | 195 | 215 | - |

The table shows the lengths, in km, of potential rail routes between six towns, A, B, C, D, E and F
a) Use Prim's algorithm, starting from A, to find a minimum spanning tree for this table. You must list the arcs that form your tree in the order that they are selected.
b) Draw your tree using the vertices given in Diagram 1 in the answer book.
(c) State the total weight of your tree.

## Instructions to Candidates

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In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper
Answer ALL the questions.
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$\qquad$

The numbers in the list above represent the lengths, in metres, of ten lengths of fabric. They are to be cut from rolls of fabric of length 60 m .
(a) Calculate a lower bound for the number of rolls needed.
(b) Use the first-fit bin packing algorithm to determine how these ten lengths can be cut from rolls of length 60 m .
(c) Use full bins to find an optimal solution that uses the minimum number of rolls.
3.


Miri
Jessie
Edward
Katie
Hegg
Beth
Louis
Philip
Natsuko
Dylan
(a) Use the quick sort algorithm to sort the above list into alphabetical order.
(b) Use the binary search algorithm to locate the name Louis.

Figure 1 shows the possible allocations of six workers, Charlotte (C), Eleanor (E), Harry (H), Matt (M), Rachel (R) and Simon (S) to six tasks, 1, 2, 3, 4, 5 and 6 .

Figure 2 shows an initial matching.
(a) List an alternating path, starting at H and ending at 4. Use your path to find an improved matching. List your improved matching.
(b) Explain why it is not possible to find a complete matching.

Simon (S) now has task 3 added to his possible allocation.
(c) Taking the improved matching found in (a) as the new initial matching, use the maximum matching algorithm to find a complete matching. List clearly the alternating path you use and your complete matching.
5.


Figure 3
[The total weight of the network is 625 m ]
Figure 3 models a network of paths in a park. The number on each arc represents the length, in m , of that path.
Rob needs to travel along each path to inspect the surface. He wants to minimise the length of his route.
(a) Use the route inspection algorithm to find the length of his route. State the arcs that should be repeated. You should make your method and working clear.

The surface on each path is to be renewed. A machine will be hired to do this task and driven along each path.
The machine will be delivered to point $G$ and will start from there, but it may be collected from any point once the task is complete.
(b) Given that each path must be traversed at least once, determine the finishing point so that the length of the route is minimised. Give a reason for your answer and state the length of your route.
6.


Figure 4
Figure 4 represents a network of roads. The number on each arc gives the length, in km, of that road.
(a) Use Dijkstra's algorithm to find the shortest distance from A to I. State your shortest route.
(b) State the shortest distance from A to G.
7. Rose makes hanging baskets which she sells at her local market. She makes two types, large and small. Rose makes $x$ large baskets and $y$ small baskets.

Each large basket costs $£ 7$ to make and each small basket costs $£ 5$ to make. Rose has $£ 350$ she can spend on making the baskets.
(a) Write down an inequality, in terms of $x$ and $y$, to model this constraint.

Two further constraints are

$$
\begin{aligned}
& y \leqslant 20 \text { and } \\
& y \leqslant 4 x .
\end{aligned}
$$

(b) Use these two constraints to write down statements that describe the numbers of large and small baskets that Rose can make.
(c) On the grid provided, show these three constraints and $x \geqslant 0, y \geqslant 0$. Hence label the feasible region, R

Rose makes a profit of $£ 2$ on each large basket and $£ 3$ on each small basket. Rose wishes to maximise her profit, $£ \mathrm{P}$.
(d) Write down the objective function.
(e) Use your graph to determine the optimal numbers of large and small baskets Rose should make, and state the optimal profit
8.


Figure 5
A construction project is modelled by the activity network shown in Figure 5. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires one worker. The project is to be completed in the shortest possible time.
(a) Complete Diagram 2 in the answer book, showing the early and late event times.
(b) State the critical activities.
(c) Find the total float for activities M and H . You must make the numbers you use in your calculations clear.
(d) On the grid provided, draw a cascade (Gantt) chart for this project.

An inspector visits the project at 1 pm on days 16 and 31 to check the progress of the work.
(e) Given that the project is on schedule, which activities must be happening on each of these days?

## 6689/01 <br> Edexcel GCE

Decision Mathematics D1
Advanced/Advanced Subsidiary
Friday 15 January 2010 - Afternoon
Time: 1 hour 30 minutes

## Materials required for examination Items included with question papers D1 Answer Book

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1.


Figure 1
Figure 1 shows the possible allocation of six people, Alice (A), Brian (B), Christine (C), David (D), Elizabeth (E) and Freddy (F), to six tasks, 1, 2, 3, 4, 5 and 6

An initial matching is Alice to task 1, Christine to task 3, David to task 4 and Elizabeth to task 5.
a) Show this initial matching on Diagram 1 in the answer book
(b) Starting from this initial matching, use the maximum matching algorithm to find a complete matching. List clearly the alternating paths that you use, and give your final matching.
2. Prim's algorithm finds a minimum spanning tree for a connected graph.
(a) Explain the terms
(i) connected graph,
(ii) tree,
(iii) spanning tree.
(b) Name an alternative algorithm for finding a minimum spanning tree.

|  |  | $\begin{aligned} & \text { O} \\ & \text { O } \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { त } \\ & \text { 3 } \\ & 0 \\ & \text { Z } \end{aligned}$ | $\begin{aligned} & \frac{0}{0} \\ & \frac{0}{x} \end{aligned}$ | 5 0 0 0 0 |  | 会 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cambridge (C) | - | 60 | 62 | 81 | 132 | 139 | 156 |
| London (L) | 60 | - | 116 | 56 | 74 | 88 | 211 |
| Norwich (N) | 62 | 116 | - | 144 | 204 | 201 | 181 |
| Oxford (O) | 81 | 56 | 144 | - | 84 | 63 | 184 |
| Portsmouth (P) | 132 | 74 | 204 | 84 | - | 43 | 269 |
| Salisbury (S) | 139 | 88 | 201 | 63 | 43 | - | 248 |
| York (Y) | 156 | 211 | 181 | 184 | 269 | 248 | - |

Figure 2
Figure 2 shows the distances by road, in miles, between seven cities.
(c) (i) Use Prim's algorithm, starting at London, to find the minimum spanning tree for these cities. You must clearly state the order in which you selected the edges of your tree, and the weight of the final tree.
(ii) Draw your tree using the vertices given in Diagram 2 in the answer book.
4. A builder is asked to replace the guttering on a house. The lengths needed, in metres, are

$$
0.6,4.0,2.5,3.2,0.5,2.6,0.4,0.3,4.0 \text { and } 1.0
$$

Guttering is sold in 4 m lengths.
(a) Carry out a quick sort to produce a list of the lengths needed in descending order. You should show the result of each pass and identify your pivots clearly.
(b) Apply the first-fit decreasing bin-packing algorithm to your ordered list to determine the total number of 4 m lengths needed.
(c) Does the answer to part (b) use the minimum number of 4 m lengths? You must justify your answer.

An algorithm is described by the flowchart shown in Figure 4
(a) Given that $\mathrm{S}=25000$, complete the table in the answer book to show the results obtained at each step when the algorithm is applied.

This algorithm is designed to model a possible system of income tax, T, on an annual salary, $£$ S.
(b) Write down the amount of income tax paid by a person with an annual salary of $£ 25000$.
(c) Find the maximum annual salary of a person who pays no tax.
6.


Figure 5 is the activity network relating to a building project. The number in brackets on each arc gives the time taken, in days, to complete the activity.
(a) Explain the significance of the dotted line from event (2) to event (3).
(b) Complete the precedence table in the answer booklet.
(c) Calculate the early time and the late time for each event, showing them on the diagram in the answer booklet.
(4)
(d) Determine the critical activities and the length of the critical path
(e) On the grid in the answer booklet, draw a cascade (Gantt) chart for the project.
7. You are in charge of buying new cupboards for a school laboratory.

The cupboards are available in two different sizes, standard and large
The maximum budget available is $£ 1800$. Standard cupboards cost $£ 150$ and large cupboards cost £300.
Let $x$ be the number of standard cupboards and $y$ be the number of large cupboards.
(a) Write down an inequality, in terms of $x$ and $y$, to model this constraint.

The cupboards will be fitted along a wall 9 m long. Standard cupboards are 90 cm long and large cupboards are 120 cm long.
(b) Show that this constraint can be modelled by

$$
3 x+4 y \leqslant 30 .
$$

You must make your reasoning clear.

Given also that $y \geqslant 2$,
(c) explain what this constraint means in the context of the question

The capacity of a large cupboard is $40 \%$ greater than the capacity of a standard cupboard. You wish to maximise the total capacity.
(d) Show that your objective can be expressed as

$$
\begin{equation*}
\text { maximise } 5 x+7 y \tag{2}
\end{equation*}
$$

(e) Represent your inequalities graphically, on the axes in your answer booklet, indicating clearly the feasible region, R
(f) Find the number of standard cupboards and large cupboards that need to be purchased. Make your method clear.

## 6689/01 <br> Edexcel GCE

## Decision Mathematics D1 <br> Advanced/Advanced Subsidiary

Thursday 27 May 2010 - Morning
Time: 1 hour 30 minutes
1.

| Hajra <br> $(\mathrm{H})$ | Vicky <br> $(\mathrm{V})$ | Leisham <br> $(\mathrm{L})$ | Alice <br> $(\mathrm{A})$ | Nicky <br> $(\mathrm{N})$ | June <br> $(\mathrm{J})$ | Sharon <br> $(\mathrm{S})$ | Tom <br> $(\mathrm{T})$ | Paul <br> $(\mathrm{P})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

The table shows the names of nine people.
(a) Use a quick sort to produce the list of names in ascending alphabetical order.

You must make your pivots clear
(b) Use the binary search algorithm on your list to locate the name Paul.

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There are 12 pages in this question paper. The answer book has 20 pages. Any blank pages are indicated
advice to Candidates
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Answers without working may not gain full credit
2.


Figure 1

Figure 1 represents the distances, in metres, between eight vertices, A, B, C, D, E, F, G and H, in a network.
(a) Use Kruskal's algorithm to find a minimum spanning tree for the network.

You should list the arcs in the order in which you consider them. In each case, state whether you are adding the arc to your minimum spanning tree
(b) Complete Matrix 1 in your answer book, to represent the network.
(c) Starting at A, use Prim's algorithm to determine a minimum spanning tree. You must clearly state the order in which you considered the vertices and the order in which you included the arcs.
(d) State the weight of the minimum spanning tree
3.

$$
\begin{array}{lllllll}
41 & 28 & 42 & 31 & 36 & 32 & 29
\end{array}
$$

The numbers in the list represent the weights, in kilograms, of seven statues. They are to be transported in crates that will each hold a maximum weight of 60 kilograms.
(a) Calculate a lower bound for the number of crates that will be needed to transport the statues.
(b) Use the first-fit bin packing algorithm to allocate the statues to the crates.
(c) Use the full bin algorithm to allocate the statues to the crates.
(d) Explain why it is not possible to transport the statues using fewer crates than the number needed for part (c).
4.


Figure 2
[The total weight of the network is 73.3 km$]$
Figure 2 models a network of tunnels that have to be inspected. The number on each arc represents the length, in km, of that tunnel.
Malcolm needs to travel through each tunnel at least once and wishes to minimise the length of his inspection route.
He must start and finish at A.
(a) Use the route inspection algorithm to find the tunnels that will need to be traversed twice. You should make your method and working clear
(b) Find a route of minimum length, starting and finishing at A . State the length of your route.

A new tunnel, CG, is under construction. It will be 10 km long.
Malcolm will have to include the new tunnel in his inspection route.
(c) What effect will the new tunnel have on the total length of his route Justify your answer.
5.


Figure 3


Figure 4

Figure 3 shows the possible allocations of six people, Amelia, Charlie, Ellie, Gemma, Jimmy and Saskia, to six tasks, 1, 2, 3, 4, 5 and 6.
Figure 4 shows an initial matching.
(a) Use the maximum matching algorithm once to find an improved matching. You must state the alternating path used and your improved matching.
(b) Explain why a complete matching is not possible.

After training, Jimmy can be assigned to tasks 4 or 5 and Ellie to tasks $2,3,5$ or 6 .
(c) Starting with your current maximal matching, use the maximum matching algorithm to obtain a complete matching. You must state the alternating path used and your final matching.
6.


Figure 5

Figure 5 shows a network of cycle tracks within a national park. The number on each arc represents the time taken, in minutes, to cycle along the corresponding track.
(a) Use Dijkstra's algorithm to find the quickest route from S to T . State your quickest route and the time it takes.
(b) Explain how you determined your quickest route from your labelled diagram.
(c) Write down the quickest route from E to T .
$\qquad$ )
7.


## Figure 6

Keith organises two types of children's activity, 'Sports Mad' and 'Circus Fun'.
He needs to determine the number of times each type of activity is to be offered.
Let $x$ be the number of times he offers the 'Sports Mad' activity. Let $y$ be the number of times he offers the 'Circus Fun' activity.

Two constraints are

$$
\begin{array}{ll} 
& x \leqslant 15 \\
\text { and } & y>6
\end{array}
$$

These constraints are shown on the graph in Figure 6, where the rejected regions are shaded out.
(a) Explain why $y=6$ is shown as a dotted line.

Two further constraints are

$$
\text { and } \begin{aligned}
3 x & \geqslant 2 y \\
5 x+4 y & \geqslant 80
\end{aligned}
$$

$$
\text { and } \quad 5 x+4 y \geqslant 80
$$

(b) Add two lines and shading to Diagram 1 in the answer book to represent these inequalities. Hence determine the feasible region and label it R .

Each 'Sports Mad’ activity costs $£ 500$
Each 'Circus Fun' activity costs $£ 800$
Keith wishes to minimise the total cost.
(c) Write down the objective function, C , in terms of $x$ and $y$
(d) Use your graph to determine the number of times each type of activity should be offered and the total cost. You must show sufficient working to make your method clear
8.


Figure 7

A project is modelled by the activity network shown in Figure 7. The activities are represented by he arcs. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires one worker. The project is to be completed in the shortest possible time
(a) Complete Diagram 2 in the answer book to show the early and late event times.
(b) State the critical activities.
(c) On Grid 1 in the answer book, draw a cascade (Gantt) chart for this project.
(d) Use your cascade chart to determine a lower bound for the number of workers needed. You must justify your answer
(Total 11 marks)

END
TOTAL FOR PAPER: 75 MARKS

## 6689/01 <br> Edexcel GCE

## Decision Mathematics D1 <br> Advanced/Advanced Subsidiary

Wednesday 19 January 2011 - Afternoon
Time: 1 hour 30 minutes

## Materials required for examination

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for ymbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

## nstructions to Candidates

Write your answers for this paper in the D1 answer book provided
In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper
Answer ALL the questions.
When a calculator is used, the answer should be given to an appropriate degree of accuracy
Do not return the question paper with the answer book.
Information for Candidates
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2)
There are 7 questions in this question paper. The total mark for this paper is 75 .
There are 8 pages in this question paper. The answer book has 16 pages. Any blank pages are indicated

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner
Answers without working may not gain full credit
1.


Figure 1
Figure 1 shows a network of roads between eight villages, A, B, C, D, E, F, G and H. The number on each arc gives the length, in miles, of the corresponding road.
(a) Use Dijkstra's algorithm to find the shortest distance from A to H .
(b) State your shortest route.
(c) Write down the shortest route from H to C and state its length

## Turn over

2. 

23
29
$11 \quad 34$
10
$14 \quad 3$
17
The numbers represent the sizes, in megabytes (MB), of eight files.
The files are to be stored on 50 MB discs.
(a) Calculate a lower bound for the number of discs needed to store all eight files.
(b) Use the first-fit bin packing algorithm to fit the files onto the discs.
(c) Perform a bubble sort on the numbers in the list to sort them into descending order. You need only write down the final result of each pass.
(d) Use the first-fit decreasing bin packing algorithm to fit the files onto the discs.
4.


Figure 3


Figure 4

Six workers, Anthony, Beth, David, Jacob, Kantola and Miri, are to be allocated to six tasks, 1, 2, $3,4,5$ and 6 .

Figure 3 shows the possible allocations of the workers, and an initial matching is shown in Figure 4.
(a) Write down the technical name given to the type of diagram shown in Figure 3.
(b) Use the maximum matching algorithm once to find an improved matching. You must state the alternating path you use and your improved matching.

Anthony now agrees to add task 6 to his possible allocations.
(c) Starting with your improved matching, use the maximum matching algorithm to obtain a complete matching. You must state the alternating path you use and your complete matching.
5.


Figure 5
[The total weight of the network is 31.6 km ]
Figure 5 models a network of roads. The road markings on these roads are to be renewed. The number on each arc represents the length, in km, of that road. In order to renew the road markings, each road must be traversed at least once.
(a) Use the route inspection algorithm, starting and finishing at A , to find a suitable route, which should be stated. You must make your method and working clear.
(b) State the roads that must be traversed twice and the length of the route.

The machine that will be used to renew the road markings can only be delivered to D. It will start at D, but it may finish at any vertex.
Each road must still be traversed at least once.
(c) Given that the route is to be minimised, determine where the machine should finish. Give reasons to justify your answer.


Figure 6
The graph in Figure 6 is being used to solve a linear programming problem.
Two of the constraints have been drawn on the graph and the rejected regions shaded out.
(a) Write down the constraints shown on the graph.

Two further constraints are

$$
\begin{array}{rlrl} 
& x+y & \geqslant 30 \\
& \text { and } & 5 x+8 y & \leqslant 400
\end{array}
$$

(b) Add two lines and shading to Graph 1 in your answer book to represent these constraints. Hence determine the feasible region and label it R

The objective is to

$$
\text { minimise } 15 x+10 y
$$

(c) Draw a profit line on Graph 1 and use it to find the optimal solution. You must label your profit line clearly.


Figure 7
The network in Figure 7 shows the activities that need to be undertaken to complete a maintenance project. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. The numbers in circles are the events
Each activity requires one worker. The project is to be completed in the shortest possible time.
(a) Complete the precedence table for this network in the answer book.
(b) Explain why each of the following is necessary.
(i) The dummy from event 6 to event 7 .
(ii) The dummy from event 8 to event 9 .
(c) Complete Diagram 2 in the answer book to show the early and the late event times.
(d) State the critical activities.
(e) Calculate the total float on activity K. You must make the numbers used in your calculation clear.
(f) Calculate a lower bound for the number of workers needed to complete the project in the minimum time.

## 6689/01 <br> Edexcel GCE

Decision Mathematics D1
Advanced/Advanced Subsidiary
Friday 20 May 2011 - Afternoon
Time: 1 hour 30 minutes

| Materials required for examination | Items included with question papers |
| :--- | :--- |
| Nil Answer Book |  |

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration,
or have retrievable mathematical formulae stored in them. must make your method clear

## Instructions to Candidates

Write your answers for this paper in the D1 answer book provided.
In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper
Answer ALL the questions.
绪
Do not return the question paper with the answer book.
Information for Candidates
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2) There are 8 questions in this question paper. The total mark for this paper is 75
There are 12 pages in this question paper. The answer book has 20 pages. Any blank pages are indicated

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner
Answers without working may not gain full credit.
$\qquad$

## Turn over

2. 



Figure 1
(a) Define the terms
(i) tree,
(ii) minimum spanning tree
(b) Use Kruskal's algorithm to find a minimum spanning tree for the network shown in Figure 1. You should list the arcs in the order in which you consider them. In each case, state whether you are adding the arc to your minimum spanning tree
(c) Draw your minimum spanning tree using the vertices given in Diagram 1 in the answer book.
(d) State whether your minimum spanning tree is unique. Justify your answer.
3.


Figure 2
Figure 2 shows the constraints of a linear programming problem in $x$ and $y$, where $R$ is the feasible region.
(a) Write down the inequalities that form region $R$.

The objective is to maximise $3 x+y$.
(b) Find the optimal values of $x$ and $y$. You must make your method clear.
(c) Obtain the optimal value of the objective function.

Given that integer values of $x$ and $y$ are now required,
(d) write down the optimal values of $x$ and $y$
4.


Figure 3 shows the possible allocations of five workers, Adam (A), Catherine (C), Harriet (H), Josh (J) and Richard (R) to five tasks, 1, 2, 3, 4 and 5

Figure 4 shows an initial matching.
There are three possible alternating paths that start at A One of them is

$$
A-3=R-4=C-5
$$

(a) Find the other two alternating paths that start at A
(b) List the improved matching generated by using the alternating path $\mathrm{A}-3=\mathrm{R}-4=\mathrm{C}-5$.
(c) Starting from the improved matching found in (b), use the maximum matching algorithm to obtain a complete matching. You must list the alternating path used and your final matching.
6.


Figure 6 shows a network of cycle tracks. The number on each arc gives the length, in km, of that track.
(a) Use Dijkstra's algorithm to find the shortest route from A to H. State your shortest route and its length.
(b) Explain how you determined your shortest route from your labelled diagram.

The track between E and F is now closed for resurfacing and cannot be used.
(c) Find the shortest route from A to H and state its length.
7.


Figure 7
A project is modelled by the activity network shown in Figure 7. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires one worker. The project is to be completed in the shortest possible time.
(a) Complete the precedence table in the answer book.
(b) Complete Diagram 1 in the answer book, to show the early event times and late event times.
(c) State the critical activities.
(d) On the grid in your answer book, draw a cascade (Gantt) chart for this project.
(e) By considering the activities that must take place between time 7 and time 16, explain why it is not possible to complete this project with just 3 workers in the minimum time.
8. A firm is planning to produce two types of radio, type A and type B.

Market research suggests that, each week

- At least 50 type A radios should be produced.
- The number of type A radios should be between $20 \%$ and $40 \%$ of the total number of radios produced.

Each type A radio requires 3 switches and each type B radio requires 2 switches. The firm can only buy 200 switches each week.

The profit on each type A radio is $£ 15$
The profit on each type B radio is $£ 12$.
The firm wishes to maximise its weekly profit
Formulate this situation as a linear programming problem, defining your variables.
(Total 7 marks)
TOTAL FOR PAPER: 75 MARKS END

## 6689/01 <br> Edexcel GCE

## Decision Mathematics D1 <br> Advanced/Advanced Subsidiary

Friday 20 January 2012 - Afternoon
Time: 1 hour 30 minutes
$\frac{\text { Materials required for examination }}{\text { Nil }} \quad \frac{\text { Items included with question papers }}{\text { D1 Answer Book }}$

Candidates may use any calculator allowed by the regulations of the Join Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, bave retrievable mathematical formulae stored in them.

## nstructions to Candidates

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Answer ALL the questions.
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Advice to Candidates
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You should show sufficient working to make your methods clear to the Examiner
Answers without working may not gain full credit.

## Write your answers in the D1 answer book for this paper.

1. 



Figure 1
Figure 1 represents the distances, in km, between eight vertices, $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}, \mathrm{F}, \mathrm{G}$ and H in a network.
(a) Use Kruskal's algorithm to find the minimum spanning tree for the network. You should list the arcs in the order in which you consider them. In each case, state whether you are adding the arc to your minimum spanning tree
(b) Starting at A, use Prim's algorithm to find the minimum spanning tree. You must clearly state the order in which you selected the arcs of your tree.
(c) Draw the minimum spanning tree using the vertices given in Diagram 1 in the answer book
2.


Figure 2
[The weight of the network is 129 miles]
Figure 2 models a network of canals. The number on each arc gives the length, in miles, of that canal.

Brett needs to travel along each canal to check that it is in good repair. He wishes to minimise the length of his route.
(a) Use the route inspection algorithm to find the length of his route. State the arcs that should be repeated. You should make your method and working clear

A canal between B and F, of length 12 miles, is to be opened and needs to be included in Brett's inspection route.
(b) Determine if the addition of this canal will increase or decrease the length of Brett's minimum route. You must make your reasoning clear
3.


Figure 3


Figure 4

Define the terms
(a) bipartite graph,
(b) matching

Figure 3 shows the possible allocations of six people, Charles (C), Emily (E), George (G), Harriet (H), Jack (J) and Shen (S), to six tasks, 1, 2, 3, 4, 5 and 6

Figure 4 shows an initial matching.
(c) Starting from this initial matching, use the maximum matching algorithm to find an improved matching. You should list the alternating path you use and state your improved matching.

Emily has task 5 added to her possible allocations and Harriet has task 3 added to her possible allocations.
(d) Starting from the improved matching found in part (c), use the maximum matching algorithm to find a complete matching. You should list the alternating path you use and state your improved matching.
5.
(a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 20.
(b) The list of numbers is to be sorted into descending order. Use a bubble sort to obtain the sorted list, giving the state of the list after each complete pass.
(c) Apply the first-fit decreasing bin packing algorithm to your ordered list to pack the numbers into bins of size 20
(3)
(d) Determine whether your answer to (c) uses the minimum number of bins. You must justify your answer
6.


Figure 6

Edgar has recently bought a field in which he intends to plant apple trees and plum trees. He can use linear programming to determine the number of each type of tree he should plant. Let $x$ be the number of apple trees he plants and $y$ be the number of plum trees he plants.

Two of the constraints are

$$
\begin{aligned}
& x \geqslant 40 \\
& y \leqslant 50
\end{aligned}
$$

These are shown on the graph in Figure 6, where the rejected region is shaded out.
(a) Use these two constraints to write down two statements that describe the number of apple trees and plum trees Edgar can plant
(1)

Two further constraints are

$$
\begin{aligned}
3 x+4 y & \leqslant 360 \\
x & \leqslant 2 y
\end{aligned}
$$

(b) Add two lines and shading to Diagram 1 in your answer book to represent these inequalities. Hence determine the feasible region and label it R .

Edgar will make a profit of $£ 60$ from each apple tree and $£ 20$ from each plum tree. He wishes to maximise his profit, P
(c) Write down the objective function.
(d) Use an objective line to determine the optimal point of the feasible region, R. You must make your method clear
(e) Find Edgar's maximum profit.
7.


Figure 7
A project is modelled by the activity network shown in Figure 7. The activities are represented by the arcs. The number in brackets on each arc gives the time required, in hours, to complete the activity. The numbers in circles are the event numbers. Each activity requires one worker.
(a) Explain the significance of the dummy activity
(i) from event 4 to event 6 ,
(ii) from event 5 to event 7
(b) Calculate the early time and the late time for each event. Write these in the boxes in the answer book.
(c) Calculate the total float on each of activities D and G. You must make the numbers you use in your calculations clear.
(d) Calculate a lower bound for the minimum number of workers required to complete the project in the minimum time.
(e) On the grid in your answer book, draw a cascade (Gantt) chart for this project.

## 6689/01

## Edexcel GCE

Decision Mathematics D1
Advanced/Advanced Subsidiary
Friday 18 May 2012 - Morning
Time: 1 hour 30 minutes
$\frac{\text { Materials required for examination }}{\text { Nil }} \quad \frac{\text { Items included with question papers }}{\text { D1 Answer Book }}$

Candidates may use any calculator allowed by the regulations of the Join Council for Qualifications. Calculators must not have the facility for ymbolic algebra manipulation or symbolic differentiation/integration,
symbolic algebra manipulation or symbolic differentiation/it

Instructions to Candidates
Write your answers for this paper in the D1 answer book provided
In the boxes on the answer book, write your centre number, candidate number, your surname, initials
and signature.
Check that you have the correct question paper
Answer ALL the questions.
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Information for Candidates
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The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2)
There are 7 questions in this question paper. The total mark for this paper is 75
There are 12 pages in this question paper. The answer book has 16 pages. Any blank pages are indicated

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner
Answers without working may not gain full credit

1. A carpet fitter needs the following lengths, in metres, of carpet.

He cuts them from rolls of length 50 m
(a) Calculate a lower bound for the number of rolls he needs. You must make your method clear
(b) Use the first-fit bin packing algorithm to determine how these lengths can be cut from rolls of length 50 m .
(c) Carry out a bubble sort to produce a list of the lengths needed in descending order. You need only give the state of the list after each pass.
(d) Apply the first-fit decreasing bin packing algorithm to show how these lengths may be cut from the rolls.
2.


Figure 1


Figure 2

Figure 1 shows the possible allocations of five workers, Charles (C), David (D), Ellie (E), Freya (F) and Georgi (G), to five tasks, 1, 2, 3, 4 and 5.

Figure 2 shows an initial matching.
(a) Starting from this initial matching, use the maximum matching algorithm to find a complete matching. State clearly the alternating path that you use and list your final matching.
(b) Find another solution starting from the given initial matching. You should state the alternating path and list the complete matching it gives
3.

|  | A | B | C | D | E | F | G |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | - | 15 | 19 | - | 22 | 24 | - |
| B | 15 | - | - | 8 | 13 | - | - |
| C | 19 | - | - | 12 | - | 16 | - |
| D | - | 8 | 12 | - | 10 | - | 18 |
| E | 22 | 13 | - | 10 | - | 15 | 16 |
| F | 24 | - | 16 | - | 15 | - | 17 |
| G | - | - | - | 18 | 16 | 17 | - |

The table shows the lengths, in km, of a network of roads between seven villages, A, B, C, D, E, F and G.
(a) Complete the drawing of the network in Diagram 1 of the answer book by adding the necessary arcs from vertex D together with their weights.
(b) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the arcs in the order that you consider them. In each case, state whether you are adding the arc to your minimum spanning tree.
(c) Draw the minimum spanning tree using the vertices provided in Diagram 2 in the answer book.
(d) State the weight of the minimum spanning tree.
4.


Figure 3
[The total weight of the network is 1436 m ]
(a) Explain the term valency

Figure 3 models a system of underground pipes. The number on each arc represents the length, in metres, of that pipe

Pressure readings indicate that there is a leak in the system and an electronic device is to be used to inspect the system to locate the leak. The device will start and finish at A and travel along each pipe at least once. The length of this inspection route needs to be minimised.
(b) Use the route inspection algorithm to find the pipes that will need to be traversed twice. You should make your method and working clear.
(c) Find the length of the inspection route

Pipe HI is now found to be blocked; it is sealed and will not be replaced. An inspection route is now required that excludes pipe HI . The length of the inspection route must be minimised.
(d) Find the length of the minimum inspection route excluding HI. Justify your answer.
(e) Given that the device may now start at any vertex and finish at any vertex, find a minimum inspection route, excluding HI .
5.


## Figure 4

Figure 4 shows a network of roads. The number on each arc represents the length, in miles, of the corresponding road.
(a) Use Dijkstra's algorithm to find the shortest route from S to T . State your shortest route and its length.
(b) Explain how you determined your shortest route from your labelled diagram.

Due to flooding, the roads in and out of D are closed
(c) Find the shortest route from S to T avoiding D. State your shortest route and its length.
6.


Figure 5 is the activity network relating to a development project. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires one worker. The project is to be completed in the shortest possible time
(a) Complete the precedence table in the answer book.
(b) Complete Diagram 1 in the answer book to show the early event times and late event times.
(c) Calculate the total float for activity E. You must make the numbers you use in your calculation clear.
(d) Calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must show your working.
(e) Schedule the activities using the minimum number of workers so that the project is completed in the minimum time.

Let $x$ be the number of standard cars it should buy.
Let $y$ be the number of luxury cars it should buy.
Figure 6 shows three constraints, other than $x, y \geqslant 0$
Two of these are $x \geqslant 20$ and $y \geqslant 8$
(a) Write, as an inequality, the third constraint shown in Figure 6.

The company decides that at least $\frac{1}{6}$ of the cars must be luxury cars.
(b) Express this information as an inequality and show that it simplifies to

$$
5 y \geqslant x
$$

You must make the steps in your working clear

Each time the cars are hired they need to be prepared. It takes 5 hours to prepare a standard car and it takes 6 hours to prepare a luxury car. There are 300 hours available each week to prepare the cars.
(c) Express this information as an inequality.
(d) Add two lines and shading to Diagram 1 in the answer book to illustrate the constraints found in parts (b) and (c).
(e) Hence determine the feasible region and label it R .

The company expects to make $£ 80$ profit per week on each car.
It therefore wishes to maximise $\mathrm{P}=80 x+80 y$, where P is the profit per week.
(f) Use the objective line (ruler) method to find the optimal vertex, V , of the feasible region. You must clearly draw and label your objective line and the vertex V .
(g) Given that P is the expected profit, in pounds, per week, find the number of each type of car that the company should buy and the maximum expected profit

## 6689/01 <br> Edexcel GCE

## Decision Mathematics D1 Advanced/Advanced Subsidiary

Wednesday 23 January 2013 - Morning
Time: 1 hour 30 minutes
$\frac{\text { Materials required for examination }}{\text { Nil }} \quad \frac{\text { Items included with question papers }}{\text { D1 Answer Book }}$

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them

## Instructions to Candidates

Write your answers for this paper in the D1 answer book provided.
In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature
Check that you have the correct question paper.
Answer ALL the questions.
When a calculator is used, the answer should be given to an appropriate degree of accuracy. Do not return the question paper with the answer book

## Information for Candidates

ull marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2)
There are 7 questions in this question paper. The total mark for this paper is 75 .
There are 8 pages in this question paper. The answer book has 16 pages. Any blank pages are indicated.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner
Answers without working may not gain full credit.

Write your answers in the D1 answer book for this paper.
1.


## Figure 1

Hero's algorithm for finding a square root is described by the flow chart shown in Figure 1
Given that $\mathrm{N}=72$ and $\mathrm{E}=8$,
(a) use the flow chart to complete the table in the answer book, working to at least seven decimal places when necessary. Give the final output correct to seven decimal places.

The flow chart is used with $\mathrm{N}=72$ and $\mathrm{E}=-8$,
(b) describe how this would affect the output.
(c) State the value of E which cannot be used when using this flow chart.
2. (a) Starting with a list of all the letters of the alphabet in alphabetical order, demonstrate how a binary search is used to locate the letter P. In each iteration, you must make clear your pivot and the part of the list you are retaining.
(b) Find the maximum number of iterations needed to locate any particular letter of the alphabet. Justify your answer.
3.


Figure 2


Figure 3

Figure 2 shows the possible allocations of six workers, Charlie (C), George (G), Jack (J), Nurry (N), Olivia ( O ) and Rachel (R), to six tasks, 1, 2, 3, 4, 5 and 6.

Figure 3 shows an initial matching.
(a) Starting from this initial matching, use the maximum matching algorithm to find an improved matching. You should give the alternating path you use and list your improved matching.
(b) Explain why it is not possible to find a complete matching.

After training, Charlie adds task 5 to his possible allocations.
(c) Taking the improved matching found in (a) as the new initial matching, use the maximum matching algorithm to find a complete matching. Give the alternating path you use and list your complete matching
4.

(a) Explain what is meant, in a network, by the term path.
(2)

Figure 4 represents a network of canals. The number on each arc represents the length, in miles, of the corresponding canal.
(b) Use Dijkstra's algorithm to find the shortest path from S to T . State your path and its length.
(c) Write down the length of the shortest path from S to F .

Next week the canal represented by arc AB will be closed for dredging.
(d) Find a shortest path from S to T avoiding AB and state its length.
5.


## Figure 5

[The weight of the network is 379]
Figure 5 represents the roads in a highland wildlife conservation park. The vertices represent warden stations. The number on each arc gives the length, in km , of the corresponding road.

During the winter months the park is closed. It is only necessary to ensure road access to the warden stations.
(a) Use Prim's algorithm, starting at A, to find a minimum connector for the network in Figure 5. You must state the order in which you include the arcs.
(b) Given that it costs $£ 80$ per km to keep the selected roads open in winter, calculate the minimum cost of ensuring road access to all the warden stations.

At the end of winter, Ben inspects all the roads before the park re-opens. He needs to travel along each road at least once. He will start and finish at A, and wishes to minimise the length of his route.
(c) Use the route inspection algorithm to find the roads that will be traversed twice. You must make your method and working clear.
(d) Find the length of the shortest inspection route.

If Ben starts and finishes his inspection route at different warden stations, a shorter inspection route is possible.
(e) Determine the two warden stations Ben should choose as his starting and finishing points in order that his route has minimum length. Give a reason for your answer and state the length of the route.
6.


Lethna is producing floral arrangements for an awards ceremony. She will produce two types of arrangement, Celebration and Party. Let $x$ be the number of Celebration arrangements made.
Let $y$ be the number of Party arrangements made.
Figure 6 shows three constraints, other than $x, y \geqslant 0$
The rejected region has been shaded.
Given that two of the three constraints are $y \leqslant 30$ and $x \leqslant 60$,
(a) write down, as an inequality, the third constraint shown in Figure 6.

Each Celebration arrangement includes 2 white roses and 4 red roses Each Party arrangement includes 1 white rose and 5 red roses.

Lethna wishes to use at least 70 white roses and at least 200 red roses.
(b) Write down two further inequalities to represent this information
(c) Add two lines and shading to Diagram 1 in the answer book to represent these two inequalities
(d) Hence determine the feasible region and label it R .

The times taken to produce each Celebration arrangement and each Party arrangement are 10 minutes and 4 minutes respectively. Lethna wishes to minimise the total time taken to produce the arrangements.
(e) Write down the objective function, T, in terms of $x$ and $y$.
(f) Use point testing to find the optimal number of each type of arrangement Lethna should produce, and find the total time she will take.
7.


Figure 7
Figure 7 is the activity network relating to a building project. The activities are represented by the arcs. The number in brackets on each arc gives the time to complete the activity. Each activity requires one worker.

The project must be completed in the shortest possible time.
(a) Explain the reason for the dotted line from event 4 to event 6 as shown in Figure 7.
(b) Complete Diagram 1 in the answer book to show the early event times and the late event times.
(c) State the critical activities
(d) Calculate the total float for activity G. You must make the numbers you use in your calculation clear.
(e) Draw a Gantt chart for this project on the grid provided in the answer book.
(f) State the activities that must be happening at time 5.5
(g) Use your Gantt chart to determine the minimum number of workers needed to complete the project in the minimum time. You must justify your answer.

## 6689/01R <br> Edexcel GCE

## Decision Mathematics D1 Advanced/Advanced Subsidiary

Friday 17 May 2013 - Morning
Time: 1 hour 30 minutes

Materials required for examination
Nil
Items included with question papers D1 Answer Book

Candidates may use any calculator allowed by the regulations of the Join Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

## Instructions to Candidates

Write your answers for this paper in the D1 answer book provided.
In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature
Check that you have the correct question paper.
Answer ALL the questions.
When a calculator is used, the answer should be given to an appropriate degree of accuracy. Do not return the question paper with the answer book.

## Information for Candidates

Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 8 questions in this question paper. The total mark for this paper is 75 .
There are 12 pages in this question paper. The answer book has 20 pages. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner Answers without working may not gain full credit.
1.


Figure 1


Figure 2

Figure 1 shows the possible allocations of six people, A, B, C, D, E and F, to six tasks, 1, 2, 3, 4, 5 and 6.

Figure 2 shows an initial matching
(a) Starting from the given initial matching, use the maximum matching algorithm to find an improved matching. You should list the alternating path you used, and your improved matching.
(b) Explain why it is not possible to find a complete matching.
2.

|  | A | B | C | D | E | F |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | - | 85 | 110 | 160 | 225 | 195 |
| B | 85 | - | 100 | 135 | 180 | 150 |
| C | 110 | 100 | - | 215 | 200 | 165 |
| D | 160 | 135 | 215 | - | 235 | 215 |
| E | 225 | 180 | 200 | 235 | - | 140 |
| F | 195 | 150 | 165 | 215 | 140 | - |

The table shows the average journey time, in minutes, between six towns, A, B, C, D, E and F.
(a) Use Prim's algorithm, starting at A, to find a minimum spanning tree for this network. You must list the arcs that form your tree in the order in which you selected them.
b) Draw your tree using the vertices given in Diagram 1 in the answer book.
(c) Find the weight of your minimum spanning tree.

Kruskal's algorithm may also be used to find a minimum spanning tree.
(d) State three differences between Prim's algorithm and Kruskal's algorithm.
3.


A project is modelled by the activity network shown in Figure 3. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires one worker. The project is to be completed in the shortest possible time.
(a) Complete Diagram 1 in the answer book to show the early event times and late event times.
(b) Calculate the total float for activity H. You must make the numbers you use in your calculation clear.
(c) Calculate a lower bound for the number of workers needed to complete the project in the shortest possible time. Show your calculation.

Diagram 2 in the answer book shows a partly completed scheduling diagram for this project.
(d) Complete the scheduling diagram, using the minimum number of workers, so that the project is completed in the minimum time.
4.

1. $\operatorname{Sam}(\mathrm{S})$
2. Janelle (J)
3. Haoyu (H)
4. Alfie (A)
5. Alfie (A)
6. Cyrus (C)
7. Komal (K)
8. Polly (P)
9. David (D)
10. Tom (T)
11. Lydia (L)

A binary search is to be performed on the names in the list above to locate the name Lydia.
(a) Using an appropriate algorithm, rearrange the list so that a binary search can be performed, showing the state of the list after each complete iteration. State the name of the algorithm you have used.
(b) Use the binary search algorithm to locate the name Lydia in the list you obtained in (a). You must make your method clear.
5.


Figure 4
[The total weight of the network is 181 miles]

Figure 4 represents a network of power cables that have to be inspected. The number on each arc represents the length, in km, of that cable.

A route of minimum length that traverses each cable at least once and starts and finishes at A needs to be found.
(a) Use the route inspection algorithm to find the arcs that will need to be traversed twice. You must make your method and working clear.
(b) Write down a possible shortest inspection route, giving its length.

It is now decided to start and finish the inspection route at two distinct vertices. The route must still traverse each cable at least once.
(c) Determine possible starting and finishing points so that the length of the route is minimised. You must give reasons for your answer
7.


Figure 5
Figure 5 represents a network of roads. The number on each arc represents the length, in miles, of the corresponding road. A large crane is required at J and it may be transported from either $\mathrm{C}_{1}$ or C. A route of minimum length is required

It is decided to use Dijkstra's algorithm to find the shortest routes between $\mathrm{C}_{1}$ and J and between $\mathrm{C}_{2}$ and J .
(a) Explain why J, rather than $\mathrm{C}_{1}$ or $\mathrm{C}_{2}$, should be chosen as the starting vertex.
(b) Use Dijkstra's algorithm to find the shortest route needed to transport the crane. State your route and its length.

A company makes two types of garden bench, the 'Rustic' and the 'Contemporary'. The company wishes to maximise its profit and decides to use linear programming.

Let $x$ be the number of 'Rustic' benches made each week and $y$ be the number of 'Contemporary' benches made each week

The graph in Figure 6 is being used to solve this linear programming problem.
Two of the constraints have been drawn on the graph and the rejected region shaded out.
(a) Write down the constraints shown on the graph giving your answers as inequalities in terms of $x$ and $y$.

It takes 4 working hours to make one 'Rustic' bench and 3 working hours to make one 'Contemporary' bench. There are 120 working hours available in each week.
(b) Write down an inequality to represent this information.

Market research shows that 'Rustic' benches should be at most $\frac{3}{4}$ of the total benches made each
week.
(c) Write down, and simplify, an inequality to represent this information. Your inequality must have integer coefficients.
(d) Add two lines and shading to Diagram 1 in your answer book to represent the inequalities of (b) and (c). Hence determine and label the feasible region, R.

The profit on each 'Rustic' bench and each 'Contemporary’ bench is $£ 45$ and $£ 30$ respectively.
(e) Write down the objective function, P , in terms of $x$ and $y$.
(f) Determine the coordinates of each of the vertices of the feasible region and hence use the vertex method to determine the optimal point.
(g) State the maximum weekly profit the company could make.

## 6689/01 <br> Edexcel GCE

## Decision Mathematics D1 Advanced/Advanced Subsidiary

Friday 17 May 2013 - Morning
Time: 1 hour 30 minutes

> Materials required for examination $\quad$ Items included with question papers Nil Answer Book Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

## nstructions to Candidates

Write your answers for this paper in the D1 answer book provided.
In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature
Check that you have the correct question paper.
Answer ALL the questions.
When a calculator is used, the answer should be given to an appropriate degree of accuracy. Do not return the question paper with the answer book.

## Information for Candidates

Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2)
There are 7 questions in this question paper. The total mark for this paper is 75 .
There are 12 pages in this question paper. The answer book has 16 pages. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examine
Answers without working may not gain full credit.

Write your answers in the D1 answer book for this paper.
1.


Figure 1


Figure 2
2.

| 0.6 | 1.5 | 1.6 | 0.2 | 0.4 | 0.5 | 0.7 | 0.1 | 0.9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

0.3
(a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 2.
(b) The list of numbers is to be sorted into descending order. Use a quick sort to obtain the sorted list. You must make your pivots clear.
(c) Apply the first-fit decreasing bin packing algorithm to your ordered list to pack the numbers into bins of size 2 .
(d) Determine whether your answer to (c) uses the minimum number of bins. You must justify your answer.

Figure 1 shows the possible allocations of six people, Alex (A), Ben (B), Harriet (H), Izzy (I), Leo (L) and Rowan (R), to six tasks, 1, 2, 3, 4, 5 and 6.
(a) Write down the technical name given to the type of diagram shown in Figure 1.
(1)

Figure 2 shows an initial matching.
(b) Starting from the given initial matching, use the maximum matching algorithm to find a complete matching. You should list the alternating paths you use, and state your improved matching after each iteration.
3.

|  | A | B | C | D | E | F |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | - | 15 | 6 | 9 | - | - |
| B | 15 | - | 12 | - | 14 | - |
| C | 6 | 12 | - | 7 | 10 | - |
| D | 9 | - | 7 | - | 11 | 17 |
| E | - | 14 | 10 | 11 | - | 5 |
| F | - | - | - | 17 | 5 | - |

The table shows the times, in days, needed to repair the network of roads between six towns, $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}$ and F , following a flood
(a) Use Prim's algorithm, starting at A, to find the minimum connector for this network. You must list the arcs that form your tree in the order that you selected them.
(b) Draw your minimum connector using the vertices given in Diagram 1 in the answer book
(c) Add arcs from D, E and F to Diagram 2 in the answer book, so that it shows the network of roads shown by the table.
(d) Use Kruskal's algorithm to find the minimum connector. You should list the arcs in the order in which you consider them. In each case, state whether you are adding the arc to your minimum connector
(e) State the minimum time needed, in days, to reconnect the six towns.
4.


Figure 3
Figure 3 represents a network of roads. The number on each arc represents the length, in miles, of the corresponding road. Liz wishes to travel from S to T .
(a) Use Dijkstra's algorithm to find the shortest path from S to T . State your path and its length.

On a particular day, Liz must include F in her route.
(b) Find the shortest path from S to T that includes F , and state its length.
5.


## Figure 4

[The total weight of the network is 344 miles]
Figure 4 represents a railway network. The number on each arc represents the length, in miles, of that section of the railway.

Sophie needs to travel along each section to check that it is in good condition
She must travel along each arc of the network at least once, and wants to find a route of minimum length. She will start and finish at A.
(a) Use the route inspection algorithm to find the arcs that will need to be traversed twice. You must make your method and working clear.
(b) Write down a possible shortest inspection route, giving its length.
6. Harry wants to rent out boats at his local park. He can use linear programming to determine the number of each type of boat he should buy.

Let $x$ be the number of 2 -seater boats and $y$ be the number of 4 -seater boats.
One of the constraints is

$$
x+y \geqslant 90
$$

(a) Explain what this constraint means in the context of the question.

Another constraint is

$$
2 x \leqslant 3 y
$$

(b) Explain what this constraint means in the context of the question.

A third constraint is

$$
y \leqslant x+30
$$

(c) Represent these three constraints on Diagram 1 in the answer book. Hence determine, and label, the feasible region R.

Each 2 -seater boat costs $£ 100$ and each 4 -seater boat costs $£ 300$ to buy. Harry wishes to minimise the total cost of buying the boats.
(d) Write down the objective function, C , in terms of $x$ and $y$.
(e) Determine the number of each type of boat that Harry should buy. You must make your method clear and state the minimum cost.

Sophie now decides to start the inspection route at E. The route must still traverse each arc at least once but may finish at any vertex.
(c) Determine the finishing point so that the length of the route is minimised. You must give reasons for your answer and state the length of your route.
7.


Figure 5

$$
\text { [The sum of the duration of all activities is } 172 \text { days] }
$$

A project is modelled by the activity network shown in Figure 5. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires one worker. The project is to be completed in the shortest possible time.
(a) Complete Diagram 1 in the answer book to show the early event times and late event times.
(b) Calculate the total float for activity M. You must make the numbers you use in your calculation clear.
(c) For each of the situations below, explain the effect that the delay would have on the project completion date.
(i) A 2 day delay on the early start of activity P.
(ii) A 2 day delay on the early start of activity Q .
(d) Calculate a lower bound for the number of workers needed to complete the project in the shortest possible time.

Diagram 2 in the answer book shows a partly completed cascade chart for this project.
(e) Complete the cascade chart.
(f) Use your cascade chart to determine a second lower bound on the number of workers needed to complete the project in the shortest possible time. You must make specific reference to times and activities.
(g) State which of the two lower bounds found in (d) and (f) is better. Give a reason for your answer.
(Total 17 marks)

## TOTAL FOR PAPER: 75 MARKS

END

## Pearson Edexcel International Advanced Level <br> Decision Mathematics D1

Advanced/Advanced Subsidiary

Wednesday 22 January 2014 - Morning
Paper Reference
Time: 1 hour 30 minutes
WDM01/01

## D1 Answer Book <br> D1 Answer Book

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them

## nstructions

- Use black ink or ball-point pen
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).

Coloured pencils and highlighter pens must not be used.

- Fill in the boxes on the top of the answer book with your name,
centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled
- Answer the questions in the D1 answer book provided
- there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.
Do not return the question paper with the answer book.
Information
- The total mark for this paper is 75
- The marks for each question are shown in brackets
- use this as a guide as to how much time to spend on each question.


## Advice

- Read each question carefully before you start to answer it
- Try to answer every question.
- Check your answers if you have time at the end.

1. 

(a) Use the bubble sort algorithm to perform ONE complete pass towards sorting these numbers into ascending order

The original list is now to be sorted into descending order.
(b) Use a quick sort to obtain the sorted list, giving the state of the list after each complete pass. You must make your pivots clear

The numbers are to be packed into bins of size 26
(c) Calculate a lower bound for the minimum number of bins required. You must show your working
2.


Figure 1
Figure 1 represents nine buildings, A, B, C, D, E, F, G, H and I, recently bought by Newberry Enterprises. The company wishes to connect the alarm systems between the buildings to form a Enterprises. The company wishes to connect the alarm systems between the buildings to form a
single network. The number on each arc represents the cost, in pounds, of connecting the alarm systems between the buildings.
(a) Use Prim's algorithm, starting at A, to find the minimum spanning tree for this network. You must list the arcs that form your tree in the order that you select them.
(b) State the minimum cost of connecting the alarm systems in the nine buildings.

It is discovered that some alarm systems are already connected. There are connections along BC and EF, as shown in bold in Diagram 1 in the answer book. Since these already exist, it is decided to use these arcs as part of the spanning tree.
(c) (i) Use Kruskal's algorithm to find the minimum spanning tree that includes arcs BC and EF. You must list the arcs in the order that you consider them. In each case, state whether you are adding the arc to your spanning tree.
(ii) Explain why Kruskal's algorithm is a better choice than Prim's algorithm in this case.

Since arcs BC and EF already exist, there is no cost for these connections.
(d) State the new minimum cost of connecting the nine buildings.
3.


Figure 2


Figure 3

Figure 2 shows the possible allocations of six people, Beth (B), Charlie (C), Harry (H), Karam (K), Sam (S) and Theresa (T), to six tasks $1,2,3,4,5$ and 6

Figure 3 shows an initial matching
(a) Define the term 'matching'.
(b) Starting from the given initial matching, use the maximum matching algorithm to find an improved matching. You should list the alternating path that you use, and state the improved matching.

After training, a possible allocation for Harry is task 6, and an additional possible allocation for Karam is task 1 .
(c) Starting from the matching found in (b), use the maximum matching algorithm to find a complete matching. You should list the alternating path that you use, and state your complete matching.
$\qquad$
4.


## Figure 4

[The total weight of the network is 367 metres]

Figure 4 represents a network of water pipes. The number on each arc represents the length, in metres, of that water pipe.

A robot will travel along each pipe to check that the pipe is in good repair.
The robot will travel along each pipe at least once. It will start and finish at A and the total distance travelled must be minimised.
(a) Use the route inspection algorithm to find the pipes that will need to be traversed twice. You must make your method and working clear.
(b) Write down the length of a shortest inspection route

A new pipe, IJ, of length 35 m is added to the network. This pipe must now be included in a new minimum inspection route starting and finishing at A .
(c) Determine if the addition of this pipe will increase or decrease the distance the robot must travel. You must give a reason for your answer.
5.


Figure 5
Figure 5 represents a network of roads. The number on each arc represents the length, in km, of the corresponding road.
(a) Use Dijkstra's algorithm to find the shortest route from S to T . State your route and its length

The road represented by arc CE is now closed for repairs.
(b) Find two shortest routes from S to T that do not include arc CE. State the length of these routes.
(6)
6. A linear programming problem in $x$ and $y$ is described as follows.

Minimise $\quad C=2 x+5 y$
subject to

$$
x+y \geqslant 500
$$

$$
5 x+4 y \geqslant 4000
$$

$$
y \leqslant 2 x
$$

$$
y \geqslant x-250
$$

$$
x, y \geqslant 0
$$

(a) Add lines and shading to Diagram 1 in the answer book to represent these constraints. Hence determine the feasible region and label it R .
(b) Use point testing to determine the exact coordinates of the optimal point, P. You must show your working.

The first constraint is changed to $x+y \geqslant k$ for some value of $k$.
(c) Determine the greatest value of $k$ for which $P$ is still the optimal point.
7.


Figure 6
A project is modelled by the activity network shown in Figure 6. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires one worker. The project is to be completed in the shortest possible time.
(a) Complete Diagram 1 in the answer book to show the early event times and late event times.
(b) Draw a cascade (Gantt) chart for this project on Grid 1 in the answer book.
(c) Use your cascade chart to determine a lower bound for the number of workers needed to complete the project in the shortest possible time. You must make specific reference to times and activities.

The project is to be completed in the minimum time using as few workers as possible.
(d) Schedule the activities, using Grid 2 in the answer book
8. A charity produces mixed packs of posters and flyers to send out to sponsors.

Pack A contains 40 posters and 20 flyers.

## Pack B contains 30 posters and 50 flyers.

The charity must send out at least 15000 flyers.
The charity wants between $40 \%$ and $60 \%$ of the total packs produced to be Pack As
Posters cost 15 p each and flyers cost 3 p each.
The charity wishes to minimise its costs.
Let $x$ represent the number of Pack As produced, and $y$ represent the number of Pack Bs produced.
Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.
You should not attempt to solve the problem
(Total 6 marks) TOTAL FOR PAPER: 75 MARKS

## 6689/01R <br> Edexcel GCE

## Decision Mathematics D1 Advanced/Advanced Subsidiary

Thursday 12 June 2014 - Afternoon
Time: 1 hour 30 minutes
$\frac{\text { Materials required for examination }}{\text { Nil }} \frac{\text { Items included with question papers }}{\text { D1 Answer Book }}$ Candidates may use any calculator allowed by the regulations of the
Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration for symbolic algebra manipulation or symbolic different

## nstructions to Candidates

Write your answers for this paper in the D1 answer book provided.
In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature
Check that you have the correct question paper.
Answer ALL the questions.
When a calculator is used, the answer should be given to an appropriate degree of accuracy. Do not return the question paper with the answer book.

## Information for Candidates

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The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2)
There are 8 questions in this question paper. The total mark for this paper is 75 .
There are 12 pages in this question paper. The answer book has 16 pages. Any blank pages are indicated.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examine
Answers without working may not gain full credit.

## Write your answers in the D1 answer book for this paper.

1. 

$\begin{array}{lllllllll}31 & 10 & 38 & 45 & 19 & 47 & 35 & 28 & 12\end{array}$
(a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 60
(b) Carry out a quick sort to produce a list of the numbers in descending order. You should show the result of each pass and identify your pivots clearly.
(c) Use the first-fit decreasing bin packing algorithm to determine how the numbers listed can be packed into bins of size 60
(d) Determine whether the number of bins used in (c) is optimal. Give a reason for your answer.
2.


Figure 1


Figure 2

Figure 1 shows the possible allocations of five employees, Ali (A), Campbell (C), Hugo (H), anelle (J) and Polly (P), to five tasks 1, 2, 3, 4 and 5.
(a) Explain why it is not possible to find a complete matching

It is decided that one of the employees should be trained so that a complete matching becomes possible. There are only enough funds for one employee to be trained.

Two employees volunteer to undergo training. Janelle can be trained to do task 1 or Hugo can be trained to do task 5.
(b) Decide which employee, Janelle or Hugo, should undergo training. Give a reason for your answer.

You may now assume that the employee you identified in (b) has successfully undergone training. Figure 2 shows an initial matching
(c) Starting from the given initial matching, use the maximum matching algorithm to find complete matching. You should list the alternating path that you use, and state the complete matching
3.


Figure 3
Figure 3 represents a network of roads. The number on each arc represents the time taken, in minutes, to traverse each road.
(a) Use Dijkstra's algorithm to find the quickest route from S to T . State your quickest route and the time taken.

It is now necessary to include E in the route.
(b) Determine the effect that this will have on the time taken for the journey. You must state your new quickest route and the time it takes.
4.


Figure 4
[The total weight of the network is 359 cm ]
Figure 4 represents the network of sensor wires used in a medical scanner. The number on each arc represents the length, in cm , of that section of wire.

After production, each scanner is tested.
A machine will be programmed to inspect each section of wire
It will travel along each arc of the network at least once, starting and finishing at A. Its route must be of minimum length.
(a) Use the route inspection algorithm to find the length of a shortest inspection route. You must make your method and working clear.

The machine will inspect 15 cm of wire per second.
(b) Calculate the total time taken, in seconds, to test 120 scanners.

It is now possible for the machine to start at one vertex and finish at a different vertex. An inspection route of minimum length is still required.
(c) Explain why the machine should be programmed to start at a vertex with odd degree.

Due to constraints at the factory, only B or D can be chosen as the starting point and there will also be a 2 second pause between tests.
(d) Determine the new minimum total time now taken to test 120 scanners. You must state which vertex you are starting from and make your calculations clear.
5. A linear programming problem in $x$ and $y$ is described as follows.

Maximise $\quad P=2 x+3 y$
subject to

$$
\begin{aligned}
x & \geqslant 25 \\
y & \geqslant 25 \\
7 x+8 y & \leqslant 840 \\
4 y & \leqslant 5 x \\
5 y & \geqslant 3 x \\
x, y & \geqslant 0
\end{aligned}
$$

(a) Add lines and shading to Diagram 1 in the answer book to represent these constraints. Hence determine the feasible region and label it R .
(b) Use the objective line method to find the optimal vertex, V, of the feasible region. You must clearly draw and label your objective line and the vertex V .
(c) Calculate the exact coordinates of vertex V .

Given that an integer solution is required,
(d) determine the optimal solution with integer coordinates. You must make your method clear.
(2)
6. (i) Draw the activity network described in the precedence table below, using activity on arc and the minimum number of dummies.

| Activity | Immediately preceding <br> activities |
| :---: | :---: |
| $A$ | - |
| $B$ | - |
| $C$ | - |
| $D$ | $A, C$ |
| $E$ | $B$ |
| $F$ | $E$ |
| $G$ | $D, F$ |
| $H$ | $D, F$ |
| $I$ | $H, I$ |
| $J$ |  |

(ii) Explain why each of your dummies is necessary.
7.


A project is modelled by the activity network shown in Figure 5. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires one worker. The project is to be completed in the shortest possible time
(a) Complete Diagram 1 in the answer book to show the early event times and late event times.
(4)
(b) Calculate the total float for activity D. You must make the numbers you use in your calculation clear
(c) Calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must show your working.

The project is to be completed in the minimum time using as few workers as possible
(d) Schedule the activities using Grid 1 in the answer book.
8. A manufacturer of frozen yoghurt is going to exhibit at a trade fair. He will take two types of frozen yoghurt, Banana Blast and Strawberry Scream.

He will take a total of at least 1000 litres of yoghurt.
He wants at least $25 \%$ of the yoghurt to be Banana Blast. He also wants there to be at most half as much Banana Blast as Strawberry Scream.

Each litre of Banana Blast costs $£ 3$ to produce and each litre of Strawberry Scream costs $£ 2$ to produce. The manufacturer wants to minimise his costs.

Let $x$ represent the number of litres of Banana Blast and $y$ represent the number of litres of Strawberry Scream

Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients

You should not attempt to solve the problem.

## TOTAL FOR PAPER: 75 MARKS

END

| Pearson Edexcel International Advanced Level |  |
| :--- | :--- |
| Decision Mathenatics D1 <br> Advanced/Advanced Subsidiary <br> Thursday 12 June 2014 - Afternoon <br> Time: 1 hour 30 minutes <br> You must have: <br> D1 Answer Book |  |

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them

## nstructions

- Use black ink or ball-point pen
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).

Coloured pencils and highlighter pens must not be used.

- Fill in the boxes on the top of the answer book with your name,
centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the D1 answer book provided
- there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.
- Do not return the question paper with the answer book.

Information

- The total mark for this paper is 75 .
- The marks for each question are shown in brackets
- use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

2. (a) (i) Define the term complete matching.
(ii) Explain the difference between a complete matching and a maximal matching.
(3)


Figure 1


Figure 2

Figure 1 shows the possible allocations of dancing partners for the Truly Come Ballroom dancing competition. Six women, Annie (A), Bella (B), Chloe (C), Danika (D), Ella (E) and Faith (F), are to be paired with six men, Kieran (K), Lucas (L), Michael (M), Nasir (N), Oliver (O) and Paul (P).

Figure 2 shows an initial matching.
(b) Use the maximum matching algorithm once to find an improved matching. You must state the alternating path you use and list your improved matching

After dance practice, it is decided that Bella could also be paired with Kieran, and Danika could also be paired with Nasir.
(c) Starting with your improved matching from part (b), use the maximum matching algorithm to obtain a complete matching. You must state the alternating path you use and list your final matching.
(Total 9 marks)
3.


Figure 3

Figure 3 shows a network representing the time taken, in minutes, to travel by train between nine towns, A, B, C, D, E, F, G, H and T.

A train is to travel from A to T without stopping
(a) Use Dijkstra's algorithm to find the quickest route from A to T and the time taken.

At present, the train travels from A to T via F without stopping.
(b) Use your answer to (a) to find the quickest route from A to T via F and the time taken.

A train is to travel from A to T , stopping for 2 minutes at each town it passes through on its route.
(c) Explain how you would adapt the network so that you could use Dijkstra's algorithm to find the quickest route for this train. You do not need to find this route.
4. (a) State three differences between Prim's algorithm and Kruskal's algorithm for finding a minimum spanning tree


## Figure 4

[The total weight of the network is 341]
(b) Use Prim's algorithm, starting at D, to find a minimum spanning tree for the network shown in Figure 4. You must list the arcs in the order in which you select them.

Figure 4 models a network of school corridors. The number on each arc represents the length, in metres, of that corridor. The school caretaker needs to inspect each corridor in the school to check that the fire alarms are working correctly. He wants to find a route of minimum length that traverses each corridor at least once and starts and finishes at his office, D.
(c) Use the route inspection algorithm to find the corridors that will need to be traversed twice. You must make your method and working clear.

The caretaker now decides to start his inspection at G. His route must still traverse each corridor at least once but he does not need to finish at G .
(d) Determine the finishing point so that the length of his route is minimised. You must give reasons for your answer and state the length of his route.
5. Michael and his team are making toys to give to children at a summer fair. They make two type of toy, a soft toy and a craft set

Let $x$ be the number of soft toys they make and $y$ be the number of craft sets they make
Each soft toy costs $£ 3$ to make and each craft set costs $£ 5$ to make
Michael and his team have a budget of $£ 1000$ to spend on making the toys for the summer fair.
(a) Write down an inequality, in terms of $x$ and $y$, to model this constraint

Two further constraints are:

$$
\begin{gathered}
y \leqslant 2 x \\
4 y-x \geqslant 210
\end{gathered}
$$

(b) Add lines and shading to Diagram 1 in the answer book to represent all of these constraints. Hence determine the feasible region and label it R .

Michael's objective is to make as many toys as possible.
(c) State the objective function.
(d) Determine the exact coordinates of each of the vertices of the feasible region, and hence use the vertex method to find the optimal number of soft toys and craft sets Michael and his team should make. You should make your method clear.
6. (a) Draw the activity network described in this precedence table, using activity on arc and dummies only where necessary.

| Activity | Immediately preceding activities |
| :---: | :---: |
| $A$ | - |
| $B$ | - |
| $C$ | - |
| $D$ | $B, C$ |
| $E$ | $A$ |
| $F$ | $C$ |
| $G$ | $D, E$ |
| $H$ | $F, G$ |
| $I$ | $C$ |
| $J$ | $G, H$ |
| $K$ |  |

(b) Explain the possible reasons dummies may be needed in activity networks.
7.


Figure 5

A company models a project by the activity network shown in Figure 5. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires exactly one worker. The project is to be completed in the shortest possible time.
(a) Add early and late event times to Diagram 1 in the answer book.
(b) State the critical path and its length.
(c) On Diagram 2 in the answer book, construct a cascade (Gantt) chart.
(d) By using your cascade chart, state which activities must be happening at
(i) time 7.5
(ii) time 16.5

It is decided that the company may use up to 25 days to complete the project.
(e) On Diagram 3 in the answer book, construct a scheduling diagram to show how this project can be completed within 25 days using as few workers as possible.

## 6689/01 <br> Edexcel GCE

Decision Mathematics D1
Advanced/Advanced Subsidiary
Thursday 12 June 2014 - Afternoon
Time: 1 hour 30 minutes

Materials required for examination Nil

Items included with question paper D1 Answer Book Candidates may use any calculator allowed by the regulations of the Joint
Council for Qualifications. Calculators must not have the facility for ymbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.
nstructions to Candidates
Write your answers for this paper in the D1 answer book provided
In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper
Answer ALL the questions.
When a calculator is used, the answer should be given to an appropriate degree of accuracy
Do not return the question paper with the answer book.
Information for Candidates
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2)
There are 8 questions in this question paper. The total mark for this paper is 75 .
There are 12 pages in this question paper. The answer book has 20 pages. Any blank pages are indicated.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
1.

|  | E | $\begin{aligned} & \text { 俞 } \\ & \stackrel{0}{0} \\ & \text { in } \end{aligned}$ | $\begin{aligned} & E \\ & E \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { EI } \\ & \text { Hin } \end{aligned}$ |  | $\begin{aligned} & \text { U } \\ & \text { D } \\ & \text { Din } \end{aligned}$ | \% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Art (A) | - | 61 | 93 | 73 | 50 | 48 | 42 |
| Biology (B) | 61 | - | 114 | 82 | 83 | 63 | 58 |
| Chemistry (C) | 93 | 114 | - | 59 | 94 | 77 | 88 |
| Drama (D) | 73 | 82 | 59 | - | 89 | 104 | 41 |
| English (E) | 50 | 83 | 94 | 89 | - | 91 | 75 |
| French (F) | 48 | 63 | 77 | 104 | 91 | - | 68 |
| Graphics (G) | 42 | 58 | 88 | 41 | 75 | 68 | - |

The table shows the travelling times, in seconds, to walk between seven departments in a college.
(a) Use Prim's algorithm, starting at Art, to find the minimum spanning tree for the network represented by the table. You must clearly state the order in which you select the edges of your tree.
(b) Draw the minimum spanning tree using the vertices given in Diagram 1 in the answer book.
(c) State the weight of the tree.
2. (a) Draw the activity network described in the precedence table below, using activity on arc and exactly two dummies.

| Activity | Immediately preceding activities |
| :---: | :---: |
| $A$ | - |
| $B$ | - |
| $C$ | - |
| $D$ | $A, B$ |
| $E$ | $C$ |
| $F$ | $A, B$ |
| $G$ | $A, B$ |
| $H$ | $E, F$ |
| $I$ | $D$ |
| $J$ | $D, G$ |
| $K$ | $H$ |

(b) Explain why each of the two dummies is necessary.
3.


Figure 1
[The total weight of the network is 451]
Figure 1 models a network of tracks in a forest that need to be inspected by a park ranger. The number on each arc is the length, in km , of that section of the forest track

Each track must be traversed at least once and the length of the inspection route must be minimised. The inspection route taken by the ranger must start and end at vertex A.
(a) Use the route inspection algorithm to find the length of a shortest inspection route. State the arcs that should be repeated. You should make your method and working clear.
(b) State the number of times that vertex J would appear in the inspection route.

The landowner decides to build two huts, one hut at vertex K and the other hut at a different vertex. In future, the ranger will be able to start his inspection route at one hut and finish at the other. The inspection route must still traverse each track at least once.
(c) Determine where the other hut should be built so that the length of the route is minimised. You must give reasons for your answer and state a possible route and its length.
4.

|  |  | $\begin{aligned} & \ddot{0} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \text { D } \end{aligned}$ | $\begin{aligned} & \ddot{0} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| Ashley (A) | T | C | V |
| Fran (F) | V | T |  |
| Jas (J) | C | D |  |
| Ned (N) | V |  |  |
| Peter (P) | V |  |  |
| Richard (R) | G | C | K |

Six pupils, Ashley (A), Fran (F), Jas (J), Ned (N), Peter (P) and Richard (R), each wish to learn a musical instrument. The school they attend has six spare instruments; a clarinet (C), a trumpet (T), a violin (V), a keyboard (K), a set of drums (D) and a guitar (G). The pupils are asked which instruments they would prefer and their preferences are given in the table above. It is decided that each pupil must learn a different instrument and each pupil needs to be allocated to exactly one of their preferred instruments.
(a) Using Diagram 1 in the answer book, draw a bipartite graph to show the possible allocations of pupils to instruments.

Initially Ashley, Fran, Jas and Richard are each allocated to their first preference
(b) Show this initial matching on Diagram 2 in the answer book.
(c) Starting with the initial matching from (b), apply the maximum matching algorithm once to find an improved matching. You must state the alternating path you use and give your improved matching.
(d) Explain why a complete matching is not possible

Fran decides that as a third preference she would like to learn to play the guitar. Peter decides that as a second preference he would like to learn to play the drums.
(e) Starting with the improved matching found in (c), use the maximum matching algorithm to obtain a complete matching. You must state the alternating path you use and your complete matching.
5.


Figure 2
Sharon is planning a road trip from Preston to York. Figure 2 shows the network of roads that she could take on her trip. The number on each arc is the length of the corresponding road in miles.
(a) Use Dijkstra's algorithm to find the shortest route from Preston (P) to York (Y). State the shortest route and its length.

Sharon has a friend, John, who lives in Manchester (M). Sharon decides to travel from Preston to York via Manchester so she can visit John. She wishes to minimise the length of her route.
(b) State the new shortest route. Hence calculate the additional distance she must travel to visit John on this trip. You must make clear the numbers you use in your calculation.
6.

24
148
19
25
6
17
9

The numbers in the list represent the exact weights, in kilograms, of 9 suitcases. One suitcase is weighed inaccurately and the only information known about the unknown weight, $x \mathrm{~kg}$, of this suitcase is that $19<x \leqslant 23$. The suitcases are to be transported in containers that can hold a maximum of 50 kilograms.
(a) Use the first-fit bin packing algorithm, on the list provided, to allocate the suitcases to containers.
(b) Using the list provided, carry out a quick sort to produce a list of the weights in descending order. Show the result of each pass and identify your pivots clearly.
(c) Apply the first-fit decreasing bin packing algorithm to the ordered list to determine the 2 possible allocations of suitcases to containers.

After the first-fit decreasing bin packing algorithm has been applied to the ordered list, one of the containers is full.
(d) Calculate the possible integer values of $x$. You must show your working.


Figure 3

Figure 3 is the activity network for a building project. The activities are represented by the arcs The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires exactly one worker. The project is to be completed in the shortest possible time.
(b) Complete Diagram 1 in the answer book to show the early event times and the late event times
(c) State the critical activities
(d) Calculate the maximum number of days by which activity G could be delayed without affecting the shortest possible completion time of the project. You must make the numbers used in your calculation clear.
(e) Calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must show your working

The project is to be completed in the minimum time using as few workers as possible.
(f) Schedule the activities using Grid 1 in the answer book.
8.


Figure 4
The graph in Figure 4 is being used to solve a linear programming problem. The four constraints have been drawn on the graph and the rejected regions have been shaded out. The four vertices of the feasible region $R$ are labelled $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D .
(a) Write down the constraints represented on the graph.

The objective function, P , is given by

$$
\mathrm{P}=x+k y
$$

where $k$ is a positive constant.
The minimum value of the function $P$ is given by the coordinates of vertex $A$ and the maximum value of the function $P$ is given by the coordinates of vertex $D$.
(b) Find the range of possible values for $k$. You must make your method clear.
(Total 8 marks)

## TOTAL FOR PAPER: 75 MARKS

## Pearson Edexcel International Advanced Level <br> Decision Mathematics D1

Advanced/Advanced Subsidiary

Paper Reference Time: 1 hour 30 minutes

WDM01/01

## You must have: <br> D1 Answer Book

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

## Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).

Coloured pencils and highlighter pens must not be used.

- Fill in the boxes on the top of the answer book with your name,
centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the D1 answer book provided - there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy
- Do not return the question paper with the answer book.

Information

- The total mark for this paper is 75.
- The marks for each question are shown in brackets
- use this as a guide as to how much time to spend on each question.


## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1. 

|  | A | B | C | D | E | F | G | H |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | - | 9 | 8 | 13 | 17 | 11 | 12 | 10 |
| B | 9 | - | 11 | 21 | 15 | 24 | 13 | 7 |
| C | 8 | 11 | - | 20 | 23 | 17 | 17 | 15 |
| D | 13 | 21 | 20 | - | 15 | 28 | 11 | 18 |
| E | 17 | 15 | 23 | 15 | - | 31 | 23 | 30 |
| F | 11 | 24 | 17 | 28 | 31 | - | 13 | 15 |
| G | 12 | 13 | 17 | 11 | 23 | 13 | - | 23 |
| H | 10 | 7 | 15 | 18 | 30 | 15 | 23 | - |

The table represents a network that shows the time taken, in minutes, to travel by car between eight villages, $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}, \mathrm{F}, \mathrm{G}$ and H .
(a) Use Prim's algorithm, starting at A, to find a minimum spanning tree for this network. You must list the arcs that form your tree in the order in which you select them.
(b) Draw your minimum spanning tree using the vertices given in Diagram 1 in the answer book and state the weight of the tree.
(c) State whether your minimum spanning tree is unique. Justify your answer.
2.


Figure 1


Figure 2

Figure 1 shows the possible allocations of six workers, Amrit (A), Bernard (B), Cameron (C) David (D), Emily (E) and Francis (F), to six tasks, 1, 2, 3, 4, 5 and 6
(a) Explain why it is not possible to find a complete matching.

Figure 2 shows an initial matching. Starting from this initial matching,
(b) find the two alternating paths that start at C .
(c) List the two improved matchings generated by using the two alternating paths found in (b).

After training, task 5 is added to Bernard's possible allocation.
Starting from either of the two improved matchings found in (c),
(d) use the maximum matching algorithm to obtain a complete matching. You must list the additional alternating path that you use, and state the complete matching.
3.

$$
\begin{array}{llllllllll}
1.1 & 0.7 & 1.9 & 0.9 & 2.1 & 0.2 & 2.3 & 0.4 & 0.5 & 1.7
\end{array}
$$

(a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 3

The list is to be sorted into descending order.
(b) (i) Starting at the left-hand end of the list, perform one pass through the list using a bubble sort. Write down the list that results at the end of your first pass.
(ii) Write down the number of comparisons and the number of swaps performed during your first pass.
(4)

After a second pass using this bubble sort, the updated list is

$$
\begin{array}{llllllllll}
1.9 & 1.1 & 2.1 & 0.9 & 2.3 & 0.7 & 0.5 & 1.7 & 0.4 & 0.2
\end{array}
$$

(c) Use a quick sort on this updated list to obtain the fully sorted list. You must make your pivots clear.
(d) Apply the first-fit decreasing bin packing algorithm to your fully sorted list to pack the numbers into bins of size 3
4.


## Figure 3

[The total weight of the network is 100]
Figure 3 represents a network of pipes in a building. The number on each arc represents the length, in metres, of the corresponding pipe.
(a) Use Dijkstra's algorithm to find the shortest path from A to J. State your path and its length.

On a particular day Kim needs to check each pipe. A route of minimum length, which traverses each pipe at least once and starts and finishes at A, needs to be found.
(b) Use an appropriate algorithm to find the arcs that will need to be traversed twice. You must make your method and working clear.
(c) Write down a possible route, giving its length.

All the pipes directly attached to B are removed. Kim needs to check all the remaining pipes and may now start at any vertex and finish at any vertex. A route is required that excludes all those pipes directly attached to B .
(d) State all possible combinations of starting and finishing points so that the length of Kim's route is minimised. State the length of Kim's route
5.

| Activity | Immediately preceding activities |
| :---: | :---: |
| A | - |
| B | - |
| C | A |
| D | A |
| E | A, B |
| F | C |
| G | E |
| H | E |
| I | H, I |
| J | F, G |
| K |  |

(a) Draw the activity network described in the precedence table, using activity on arc. Your activity network must contain only the minimum number of dummies.
(b) Explain why, in general, dummies may be required in an activity network.
6. Jonathan is going to make hats to sell at a fete. He can make red hats and green hats. Jonathan can use linear programming to determine the number of each colour of hat that he should make.

Let $x$ be the number of red hats he makes and $y$ be the number of green hats he makes.
One of the constraints is that there must be at least 30 hats
(a) Write down an inequality, in terms of $x$ and $y$, to model this constraint.

Two further constraints are

$$
\begin{aligned}
& 2 y+x \geqslant 40 \\
& 2 y-x \geqslant-30
\end{aligned}
$$

(b) Write down two more constraints which apply
(c) Represent all these constraints on Diagram 1 in the answer book. Hence determine, and label, the feasible region $R$.

The cost of making a green hat is three times the cost of making a red hat. Jonathan wishes to minimise the total cost.
(d) Use the objective line (ruler) method to determine the number of red hats and number of green hats that Jonathan should make. You must clearly draw and label your objective line.

Given that the minimum total cost of making the hats is $£ 107.50$
(e) determine the cost of making one green hat and the cost of making one red hat. You must make your method clear.


Figure 4
[The sum of all the activity durations is 99 days]
The network in Figure 4 shows the activities that need to be undertaken to complete a project. Each activity is represented by an arc and the duration of the activity, in days, is shown in brackets. The early event times and late event times are to be shown at each vertex and some have been completed for you.

Given that activity F is a critical activity and that the total float on activity G is 2 days,
(a) write down the value of $x$ and the value of $y$,
(b) calculate the missing early event times and late event times and hence complete Diagram 1 in your answer book.

Each activity requires one worker and the project must be completed in the shortest possible time.
(c) Calculate a lower bound for the number of workers needed to complete the project in the shortest possible time.
(d) Draw a cascade (Gantt) chart for this project on Grid 1 in the answer book.
(e) Use your cascade chart to determine the minimum number of workers needed to complete the project in the shortest possible time. You must make specific reference to times and activities. (You do not need to provide a schedule of the activities.)

## Pearson Edexcel International Advanced Level Decision Mathematics D1

 Advanced/Advanced SubsidiaryTuesday 9 June 2015 - Morning

## Time: 1 hour 30 minutes

Paper Reference
WDM01/01

## You must have <br> D1 Answer Book

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

## Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B)

Coloured pencils and highlighter pens must not be used.

- Fill in the boxes on the top of the answer book with your name,
centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the D1 answer book provided
- there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy
- Do not return the question paper with the answer book.

Information

- The total mark for this paper is 75 .
- The marks for each question are shown in brackets
- use this as a guide as to how much time to spend on each question.


## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.


## Write your answers in the D1 answer book for this paper.

1. 



Figure 1
Figure 1 represents a network of roads. The number on each arc gives the length, in km, of the corresponding road.
(a) Use Dijkstra's algorithm to find the shortest distance from S to G . State the shortest route.
(6)
(b) State both the shortest distance and the shortest route from S to H
2. A list of $n$ numbers needs to be sorted into descending order starting at the left-hand end of the list.
(a) Describe how to carry out the first pass of a bubble sort on the numbers in the list.
(b) (i) State which number in the list is guaranteed to be in the correct final position after the first pass.
(ii) Write down the maximum number of passes required to sort a list of $n$ numbers.
(c) The numbers below are to be sorted into descending order. Use a bubble sort, starting at the left-hand end of the list, to obtain the sorted list. You need only give the state of the list after each pass.
$11 \quad 9$
13
7
12
8
(d) Apply the first-fit decreasing bin packing algorithm to your ordered list to pack the numbers into bins of size 21
3.


Figure 2
Figure 2 shows a graph G.
(a) Write down an example of a cycle on G.
(c) Use Prim's algorithm, starting at P , to find the minimum spanning tree for the network. You must clearly state the order in which you select the arcs of your tree.
(d) Use Kruskal's algorithm to find the minimum spanning tree for the network. You should list the arcs in the order in which you consider them. In each case, state whether you are adding the arc to the minimum spanning tree.
(e) Draw the minimum spanning tree using the vertices given in Diagram 1 in the answer book.

The weight on arc RU is now increased to a value of $x$. The minimum spanning tree for the network is still unique and includes the same arcs as those found in (e).
(f) Write down the smallest interval that must contain $x$.


Figure 3
The numbers on the 14 arcs in Figure 3 represent the distances, in km, between eight vertices, $\mathrm{P}, \mathrm{Q}$, R, S, T, U, V and W, in a network.
4. (a) Define the term 'alternating path'.
(2)


## Figure 4

Figure 4 shows the possible allocations of five people, A, B, C, D and E, to five tasks, 1, 2, 3, 4 and 5

An initial matching has three people allocated to three of the tasks.
Starting from this initial matching, one possible alternating path that starts at E is

$$
\mathrm{E}-2=\mathrm{B}-3=\mathrm{A}-4=\mathrm{D}-1
$$

(b) Use this information to
(i) deduce this initial matching,
(ii) list the improved matching generated by the given alternating path
(c) Starting from the improved matching found in (b), use the maximum matching algorithm to obtain a complete matching. You must list the alternating path you use and the final matching.
(Total 7 marks)
5.


Figure 5
[The total weight of the network is 214]
Figure 5 models a network of canals that have to be inspected. The number on each arc represents the length, in km, of the corresponding canal. Priya needs to travel along each canal at least once and wishes to minimise the length of her inspection route.

She must start and finish at A.
(a) Use the route inspection algorithm to find the length of her route. State the arcs that will need to be traversed twice. You should make your method and working clear.
(b) State the number of times that vertex F would appear in Priya's route.

It is now decided to start the inspection route at H . The route must still travel along each canal at least once but may finish at any vertex.
(c) Determine the finishing point so that the length of the route is minimised. You must give reasons for your answer and state the length of the minimum route.


Figure 6
[The sum of the durations of all the activities is 142 days]
A project is modelled by the activity network shown in Figure 6. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires one worker. The project is to be completed in the shortest possible time.
(a) Complete the precedence table in the answer book.
(b) Complete Diagram 1 in the answer book to show the early event times and late event times.
(c) Calculate the total float for activity D. You must make the numbers you use in your calculation clear.
(1)
(d) Calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must show your working.
(1)

Diagram 2 in the answer book shows a partly completed scheduling diagram for this project.
(e) Complete the scheduling diagram, using the minimum number of workers, so that the project is completed in the minimum time.
7. Ian plans to produce two types of book, hardbacks and paperbacks. He will use linear programming to determine the number of each type of book he should produce.

Let $x$ represent the number of hardbacks Ian will produce.
Let $y$ represent the number of paperbacks Ian will produce.
Each hardback takes 1 hour to print and 15 minutes to bind.
Each paperback takes 35 minutes to print and 24 minutes to bind.
The printing machine must be used for at least 14 hours. The binding machine must be used for at most 8 hours.
(a) (i) Show that the printing time restriction leads to the constraint $12 x+7 y \geqslant k$, where $k$ is a constant to be determined.
(ii) Write the binding time restriction in a similar simplified form.

Ian decides to produce at most twice as many hardbacks as paperbacks
(b) Write down an inequality to model this constraint in terms of $x$ and $y$.
(c) Add lines and shading to Diagram 1 in the answer book to represent the constraints found in (a) and (b). Hence determine, and label, the feasible region R

Ian wishes to maximise $\mathrm{P}=60 x+36 y$, where P is the total profit in pounds.
(d) (i) Use the objective line (ruler) method to find the optimal vertex, V, of the feasible region. You must draw and clearly label your objective line and the vertex V.
(ii) Determine the exact coordinates of V. You must show your working.
(e) Given that P is Ian's expected total profit, in pounds, find the number of each type of book that he should produce and his maximum expected profit.

## 6689/01

## Edexcel GCE

Decision Mathematics D1
Advanced/Advanced Subsidiary
Tuesday 9 June 2015 - Morning
Time: 1 hour 30 minutes

## Materials required for examination Items included with question papers D1 Answer Book

Candidates may use any calculator allowed by the regulations of the Join Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

## Instructions to Candidates

Write your answers for this paper in the D1 answer book provided.
In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper
Answer ALL the questions.
When a calculator is used, the answer should be given to an appropriate degree of accuracy
Do not return the question paper with the answer book.
Information for Candidates
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2)
There are 7 questions in this question paper. The total mark for this paper is 75 .
There are 12 pages in this question paper. The answer book has 20 pages. Any blank pages are indicated
Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner
Answers without working may not gain full credit
1.


Figure 1


Figure 2

A delivery firm has six vans, A, B, C, D, E and F, available for six deliveries, 1, 2, 3, 4, 5 and 6 . Each van must be assigned to just one delivery

The bipartite graph shown in Figure 1 shows the possible matchings and Figure 2 shows an initial matching.

A complete matching is required, starting from the given initial matching.
(a) Explain why it is necessary to use the maximum matching algorithm twice

There are three possible alternating paths that start at either D or B . One of these is

$$
\mathrm{D}-2=\mathrm{A}-3=\mathrm{F}-6=\mathrm{E}-5
$$

(b) Find the other two alternating paths
(c) List the improved matching generated by using the alternating path

$$
\mathrm{D}-2=\mathrm{A}-3=\mathrm{F}-6=\mathrm{E}-5
$$

(d) Starting from the improved matching found in (c), use the maximum matching algorithm to find a complete matching You should list the alternating path that you use and your complet matching
2.

$$
\begin{array}{llllllllll}
18 & 29 & 48 & 9 & 42 & 31 & 37 & 24 & 27 & 41
\end{array}
$$

The numbers above are Alan's batting scores for the first 10 cricket matches of the season.
(a) Use a quick sort to sort this list of numbers into ascending order. You must make your pivots clear.

Alan's batting scores for the final 10 cricket matches of the same season were
7253
$89 \quad 91$
$68 \quad 67$
90
$\begin{array}{lll}77 & 83 & 75\end{array}$
(b) Carry out a bubble sort on this second list of numbers to produce a list of these scores in ascending order. You need only give the state of the list after each pass.
(4)

Alan's combined batting scores for the entire season were
$\begin{array}{llllllllllllllllllll}9 & 18 & 24 & 27 & 29 & 31 & 37 & 41 & 42 & 48 & 53 & 67 & 68 & 72 & 75 & 77 & 83 & 89 & 90 & 91\end{array}$
(c) Use the binary search algorithm to locate 68 in the combined list of 20 scores. You must make your method clear

Figure 3 represents a network of roads. The number on each arc is the length, in km , of the corresponding road.
(a) Use Dijkstra's algorithm to find the shortest route from A to J. State the shortest route and its length
(b) Explain how you determined the shortest route from your labelled diagram.
(c) Find the shortest route from A to J via E and state its length.
4.


## Figure 4

[The total weight of the network is 2090]
(a) Explain why a network cannot have an odd number of vertices of odd valency.

Figure 4 represents a network of 13 roads in a village. The number on each arc is the length, in metres, of the corresponding road. A route of minimum length that traverses each road at least once needs to be found. The route may start at any vertex and finish at any vertex.
(b) Write down the vertices at which the route will start and finish

A new road, AB , of length 130 m is built. A route of minimum length that traverses each road, including AB , needs to be found. The route must start and finish at A .
(c) Use the route inspection algorithm to find the roads that will need to be traversed twice. You must make your method and working clear.
(d) Calculate the length of a possible shortest inspection route.

It is now decided to start and finish the inspection route at two distinct vertices. A route of minimum length that traverses each road, including AB , needs to be found. The route must start at A .
(e) State the finishing point so that the length of the route is minimised. Calculate how much shorter the length of this route is compared to the length of the route in (d). You must make your method and calculations clear.
5.


Figure 5
The numbers on the 17 arcs in the network shown in Figure 5 represent the distances, in km, between nine nodes, A, B, C, D, E, F, G, H and J.
(a) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the arcs in the order in which you consider them. In each case, state whether you are adding the arc to your minimum spanning tree.
(b) Starting at G, use Prim's algorithm to find a minimum spanning tree. You must clearly state the order in which you select the arcs of your tree.
(c) Find the weight of the minimum spanning tree.

A connected graph V has $n$ nodes. The sum of the degrees of all the nodes in V is $m$. The graph T is a minimum spanning tree of V .
(d) (i) Write down, in terms of $m$, the number of arcs in V
(ii) Write down, in terms of $n$, the number of arcs in T .
(iii) Hence, write down an inequality, in terms of $m$ and $n$, comparing the number of arcs in T and V .
6. A linear programming problem in $x$ and $y$ is described as follows.

Minimise $C=2 x+3 y$
subject to

$$
\begin{aligned}
x+y & \geqslant 8 \\
x & <8 \\
4 y & \geqslant x \\
3 y & \leqslant 9+2 x
\end{aligned}
$$

(a) Add lines and shading to Diagram 1 in the answer book to represent these constraints.
(b) Hence determine the feasible region and label it R.
(c) Use the objective line (ruler) method to find the exact coordinates of the optimal vertex, V , of the feasible region. You must draw and label your objective line clearly.
(d) Calculate the corresponding value of $C$ at V .

The objective is now to maximise $2 x+3 y$, where $x$ and $y$ are integers.
(e) Write down the optimal values of $x$ and $y$ and the corresponding maximum value of $2 x+3 y$.

A further constraint, $y \leqslant k x$, where $k$ is a positive constant, is added to the linear programming problem.
(f) Determine the least value of $k$ for which this additional constraint does not affect the feasible region.
7.

| Activity | Time taken (days) | Immediately preceding activities |
| :---: | :---: | :---: |
| A | 5 | - |
| B | 7 | - |
| C | 8 | - |
| D | 5 | A |
| E | 7 | A |
| F | 10 | B, C |
| G | 4 | B, C |
| H | 9 | C |
| I | 8 | G, H |
| J | 12 | G, H |
| K | 10 | D |
| L |  | E, F, I, J |

The table shows the activities required for the completion of a building project. For each activity the table shows the time taken, in days, and the immediately preceding activities. Each activity equires one worker. The project is to be completed in the shortest possible time.


Figure 6 shows a partially completed activity network used to model the project. The activities are represented by the arcs and the numbers in brackets on the arcs are the times taken, in days, to complete each activity.
(a) Add activities, E, F and I, and exactly one dummy to Diagram 1 in the answer book.
(b) Complete Diagram 1 in the answer book to show the early event times and late event times.
(c) Calculate a lower bound for the number of workers needed to complete the project in the shortest possible time. You must show your working.
(d) Schedule the activities, using the minimum number of workers, so that the project is completed in the minimum time.

# (Total 13 marks) 

TOTAL FOR PAPER: 75 MARKS
END

